

Selective Padding for Polycube-based Hexahedral Meshing

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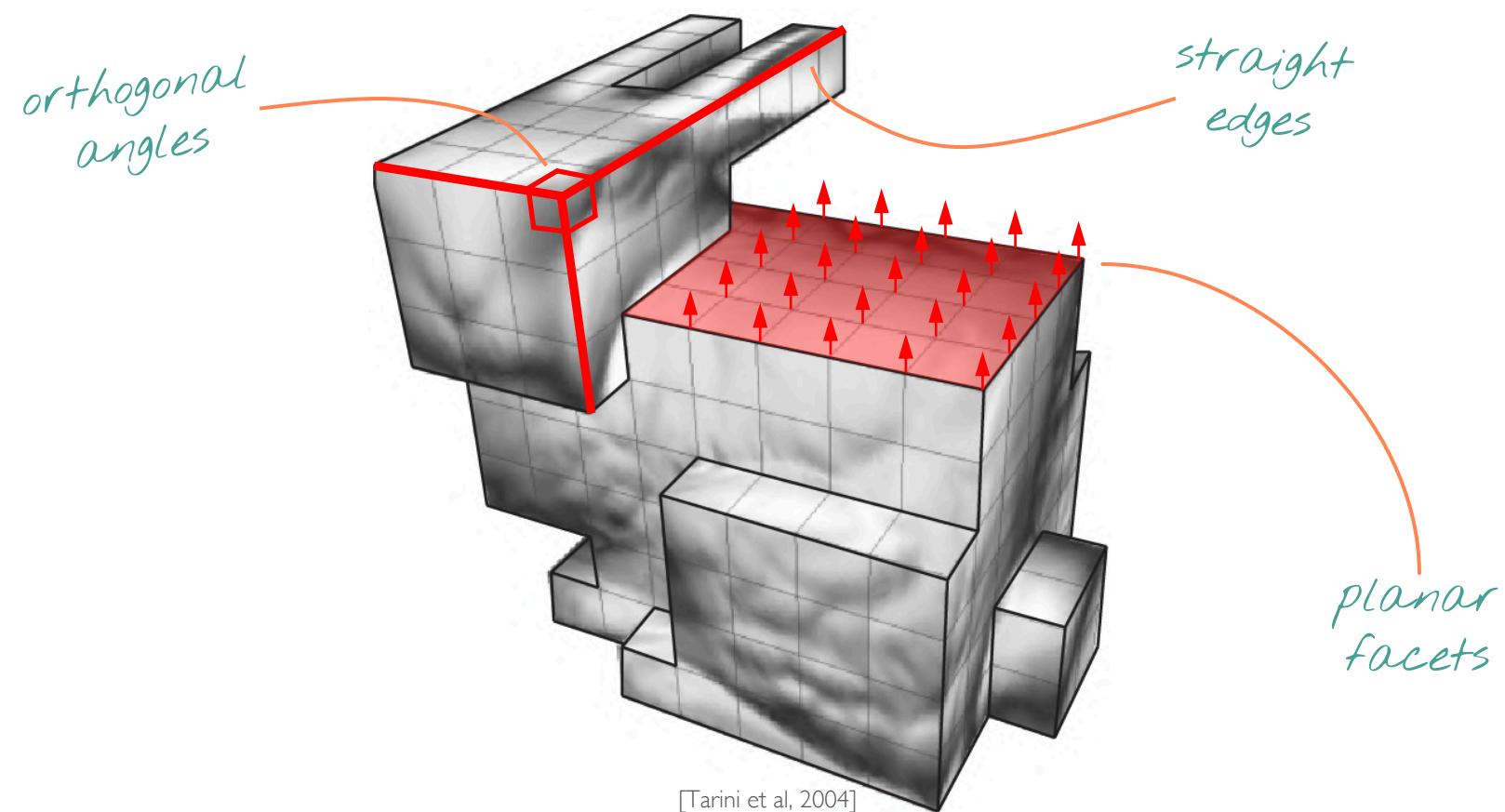
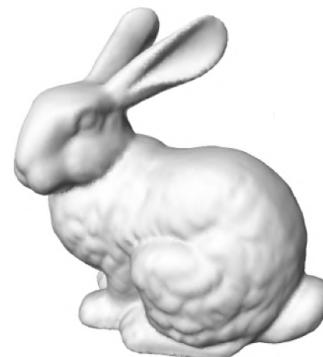
inria

RWTH u^b



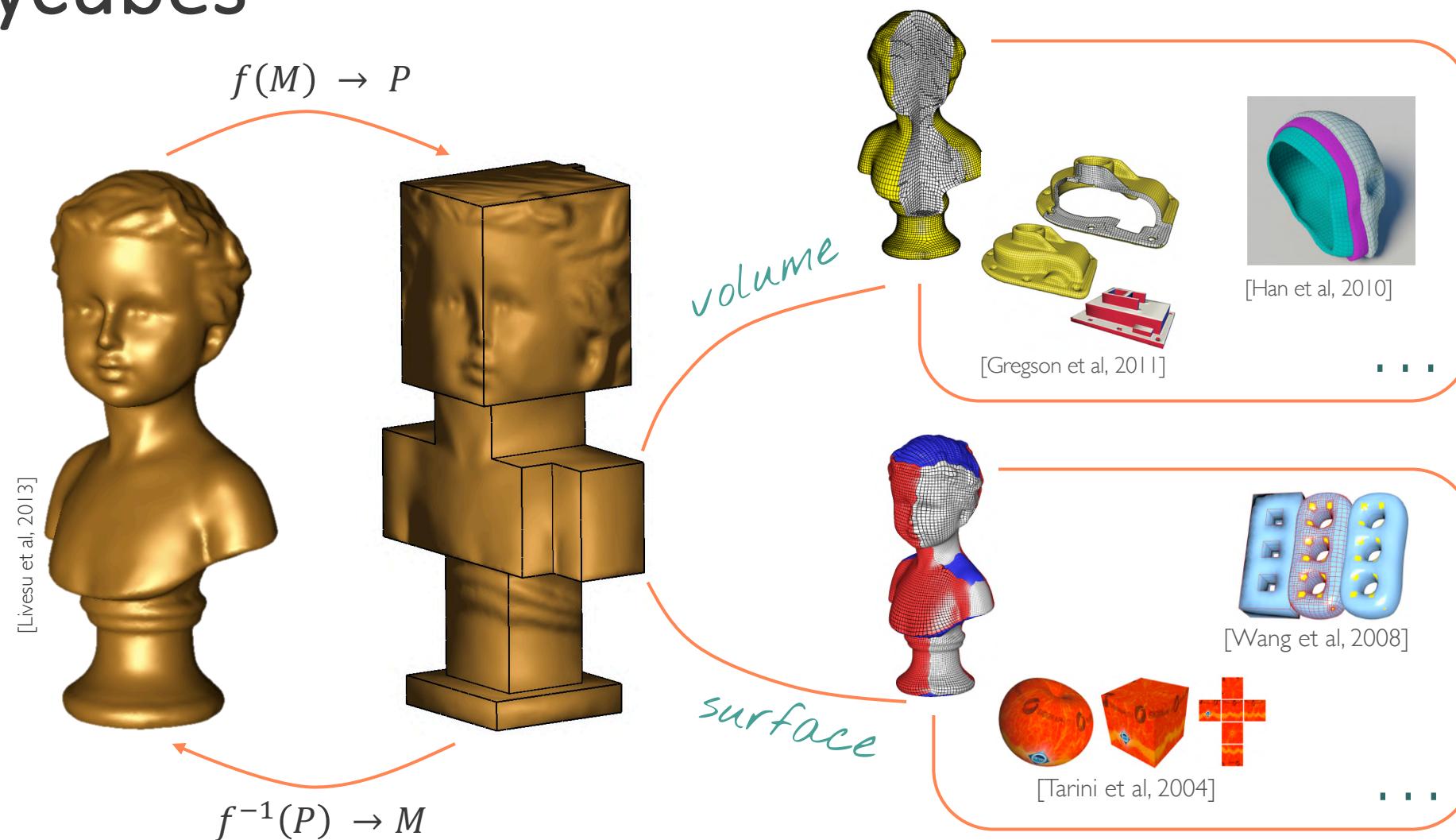
Polycubes

A polycube is a very simple representation (orthogonal polyhedra) of a tridimensional model, made up of a set of connected cuboids.

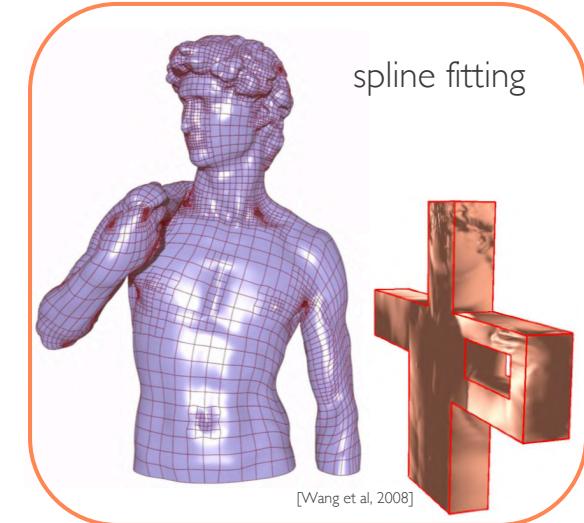
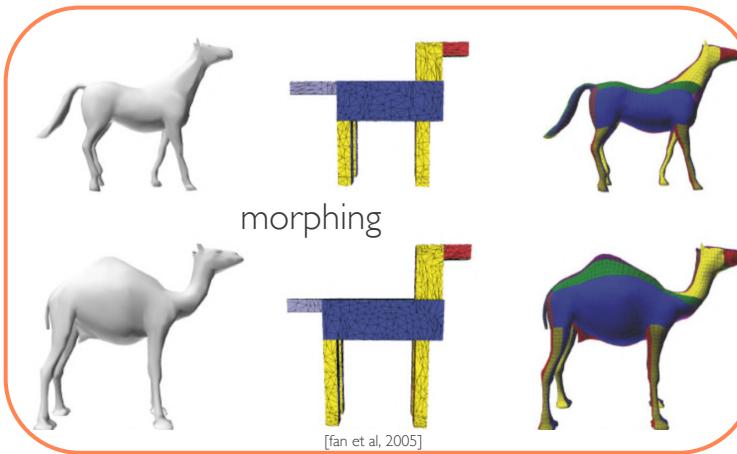
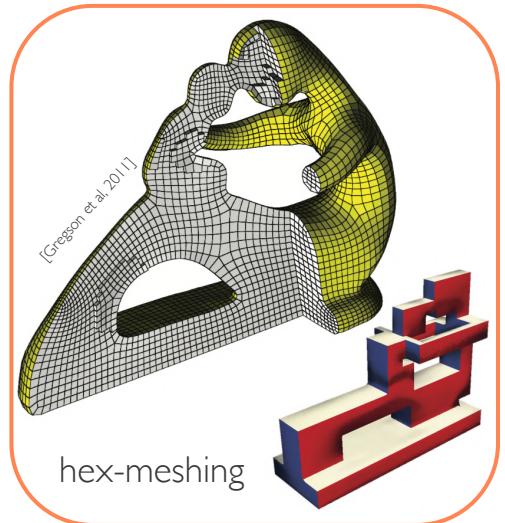


[Tarini et al, 2004]

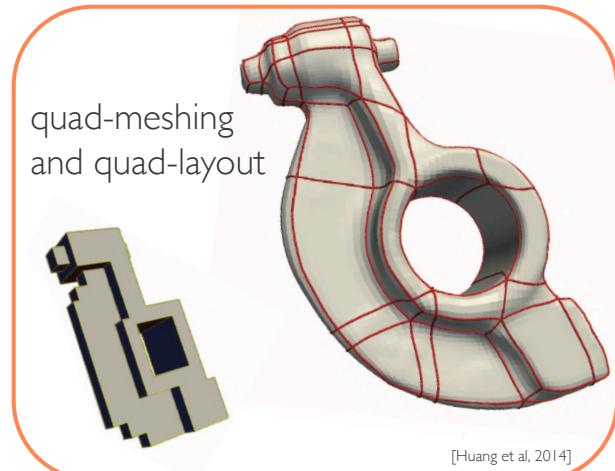
Polycubes



Why polycubes?

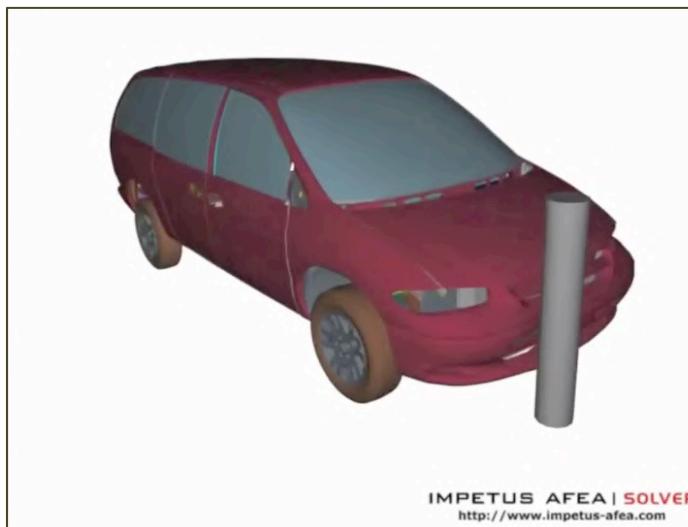


and others...

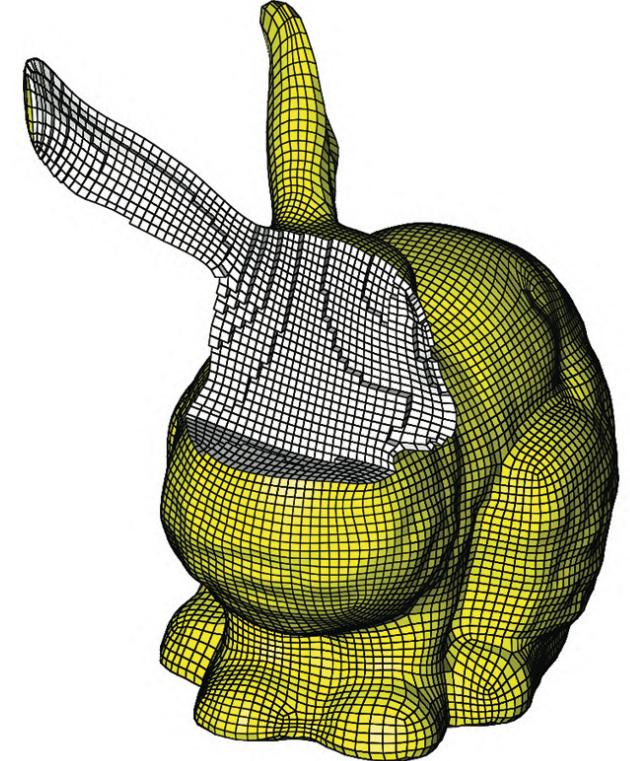


Hexahedral meshes

- A hex-mesh is a volumetric mesh where each element is a **hexahedron**
- The union of all elements is the desired volume

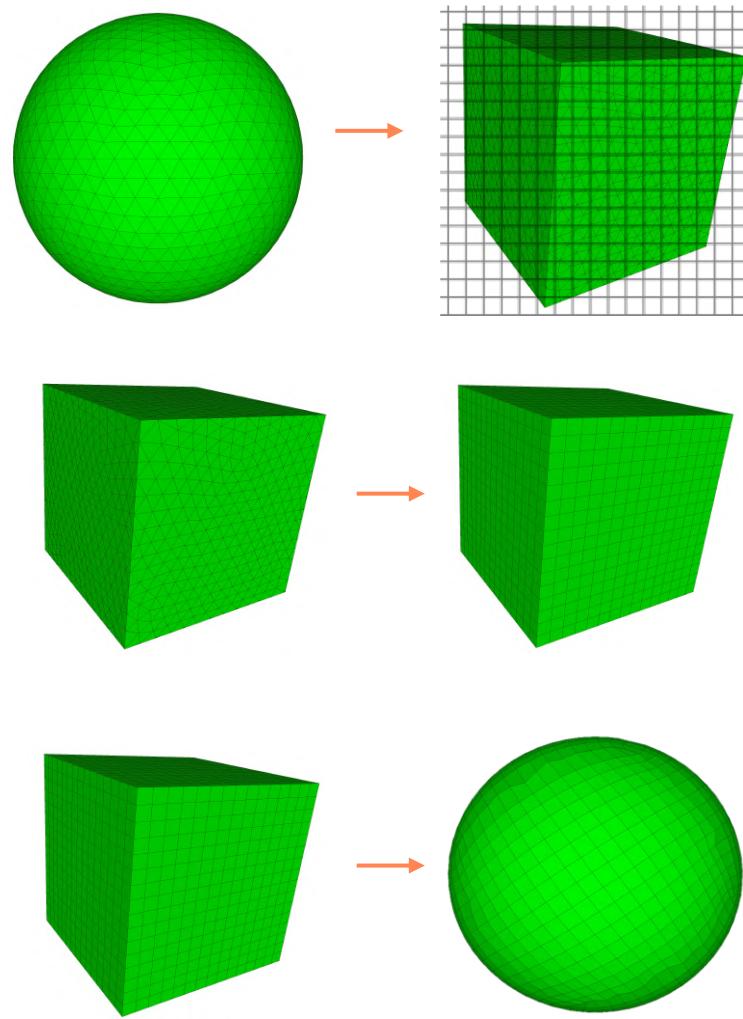


may not be planar!
may not be convex!



Polycube-based meshing pipeline

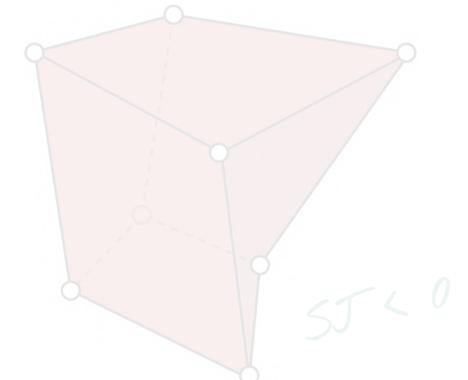
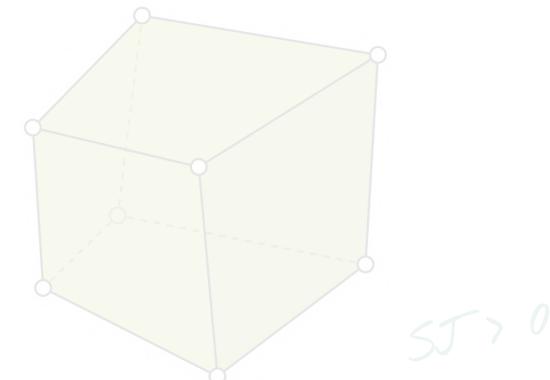
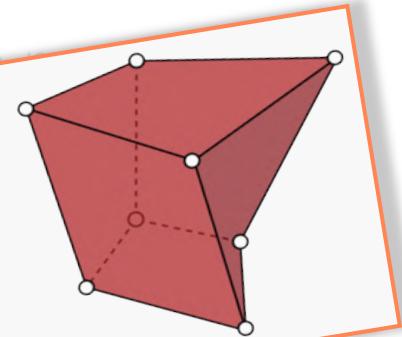
- We map the volume of the shape to the polycube space, where the generation of the hex-mesh is easier
- We define the mesh structure in such space
- We use the inverse mapping to bring the hex-mesh back to \mathbb{R}^3



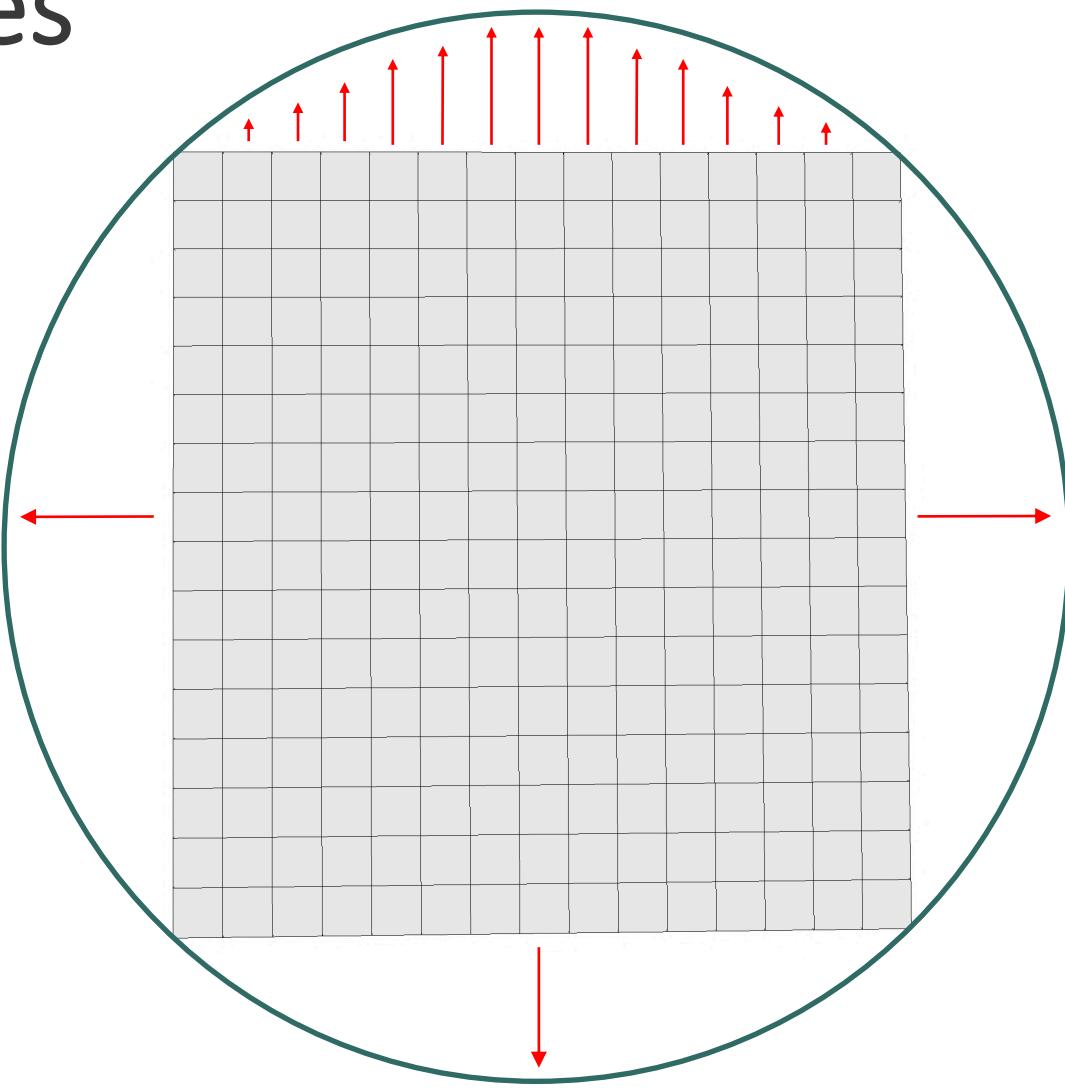
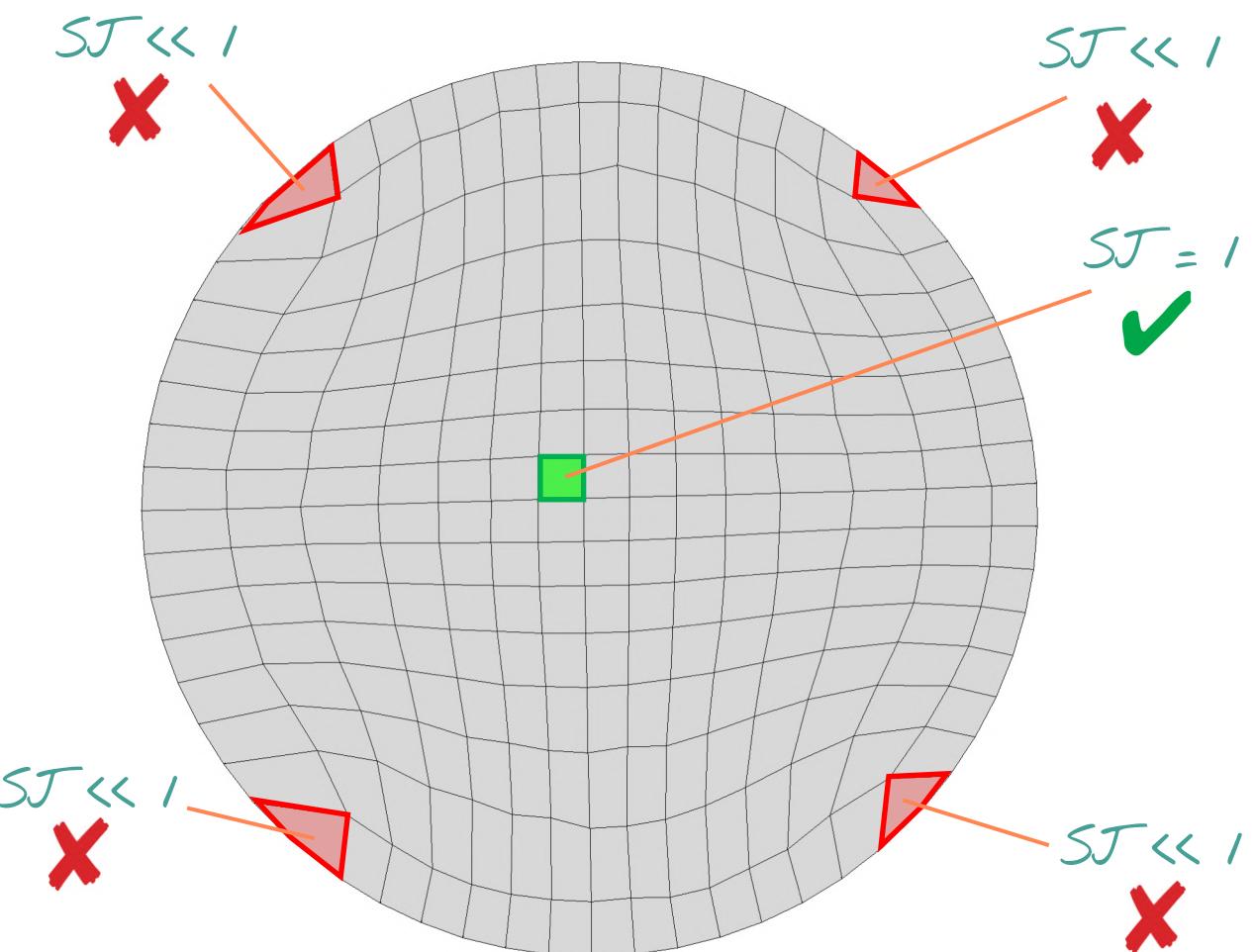
Hex-mesh Scaled Jacobian (SJ)

- It is the most popular quality metric for hex-meshes
- It measures the quality of a hexahedron, defined within the element by:
 - Ideal hexahedron = perfect cube ($SJ = 1$)
 - A hexahedron convex ($SJ > 0$)
 - A locally inverted hexahedron ($SJ < 0$)

even ONE non-convex (inverted) element makes meshes unusable!

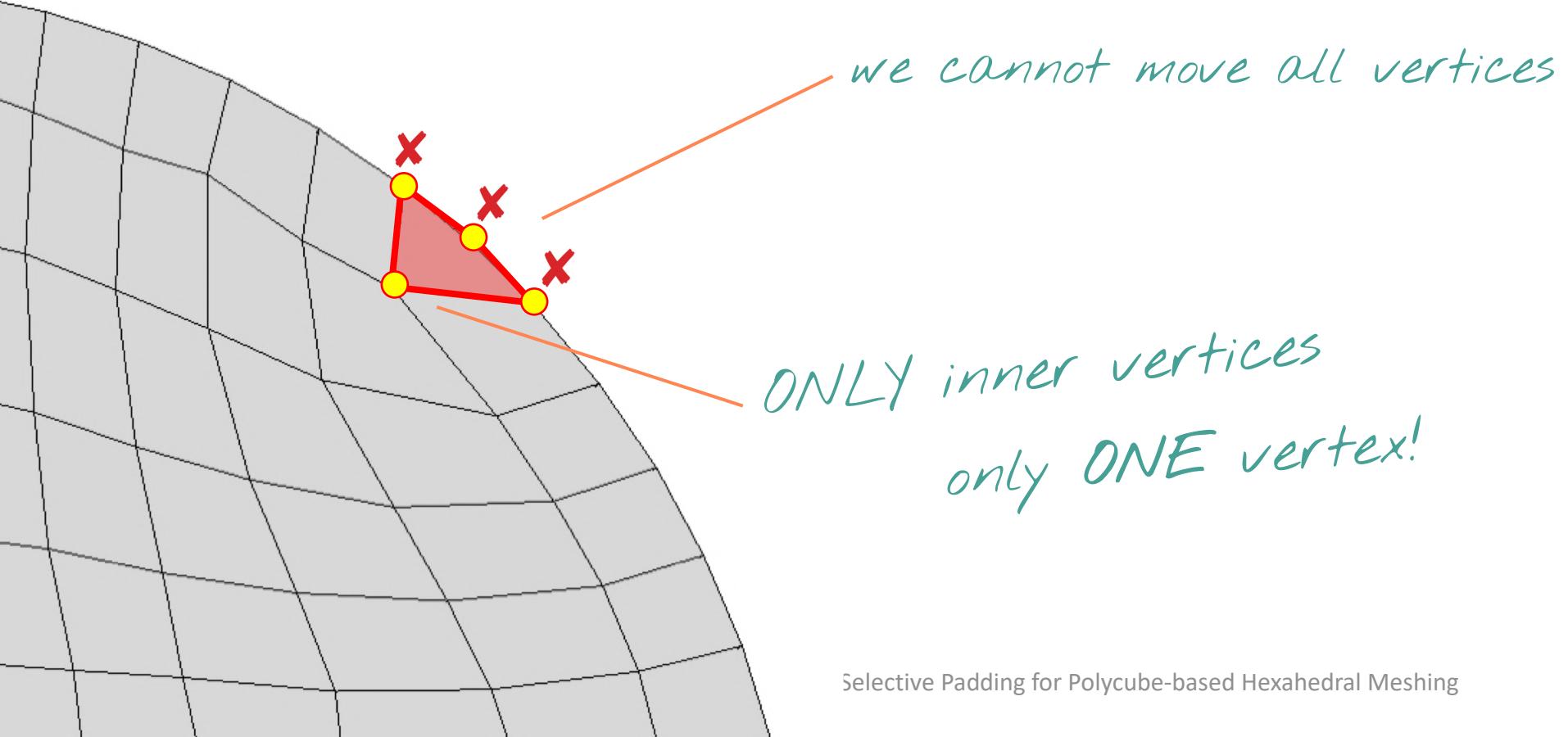


Polycube-base hex-meshes



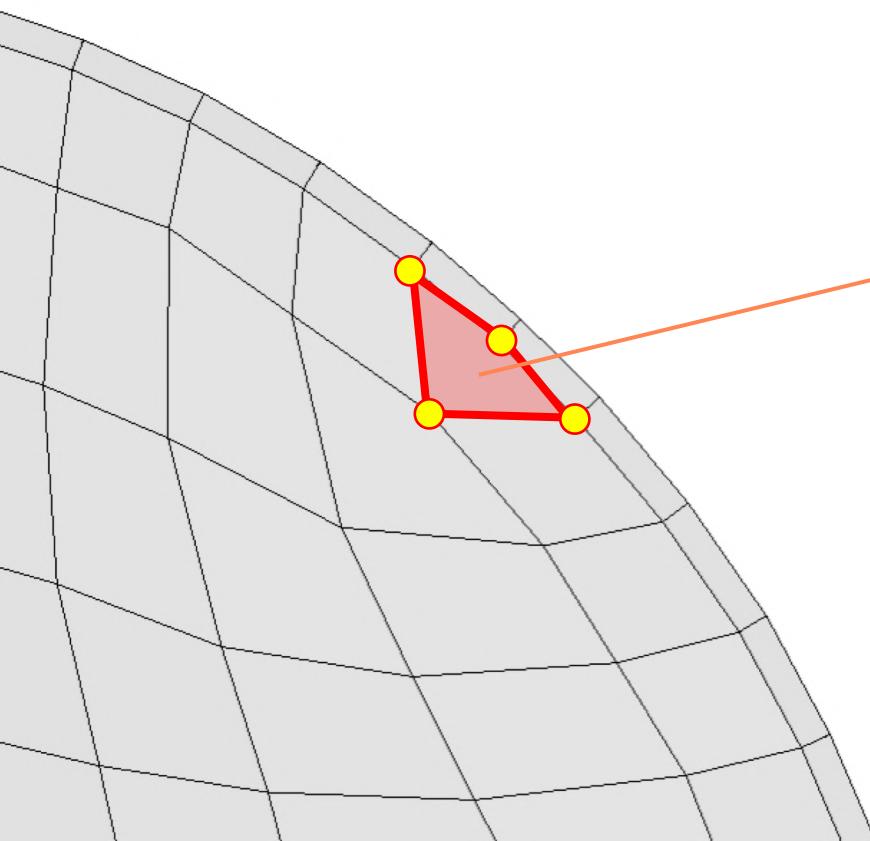
Quality improvement

- We have a problem with the degrees of freedom
 - How many vertices can we move to improve the quality?



The solution is “padding” the mesh

- We add a layer of new hexahedra in all the surface to add degrees of freedom
- Global operation **X**

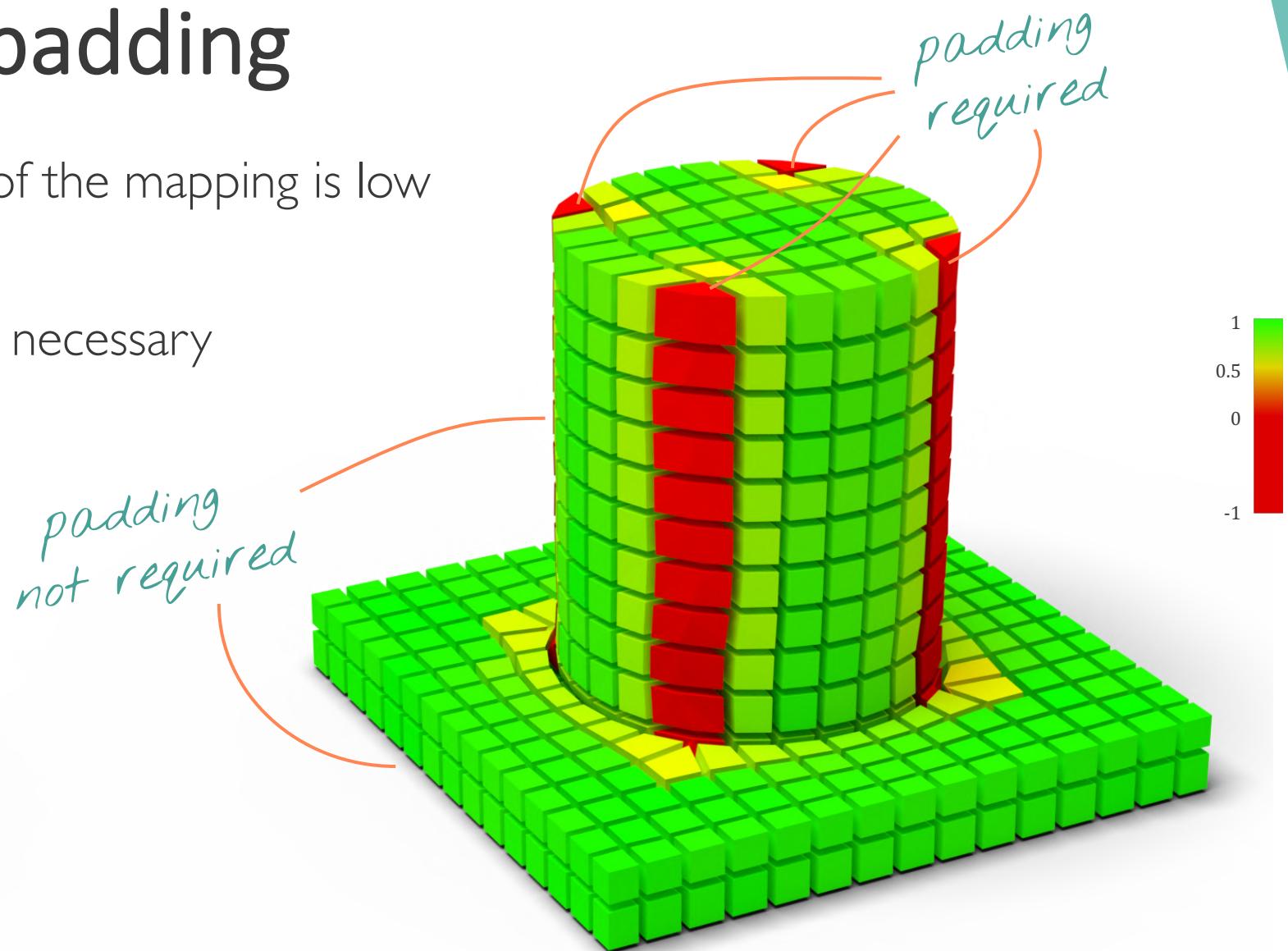


*now we can move
all vertices!*

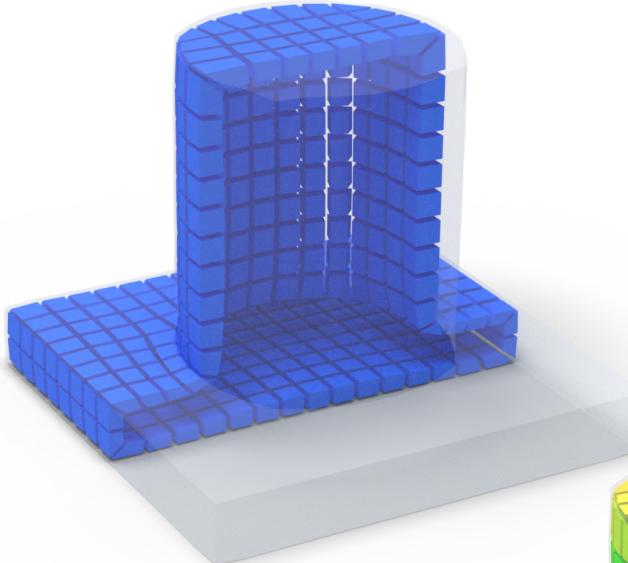


Idea: Selective padding

- Padding where the quality of the mapping is low
- No padding where it is not necessary

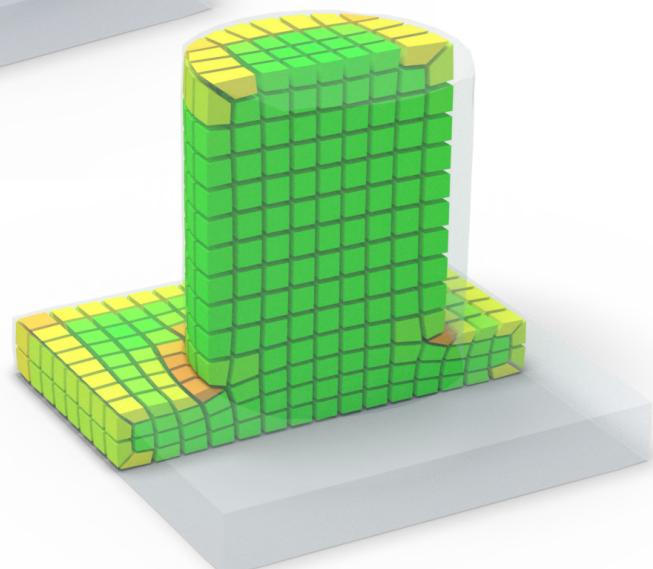


Global vs Selective padding



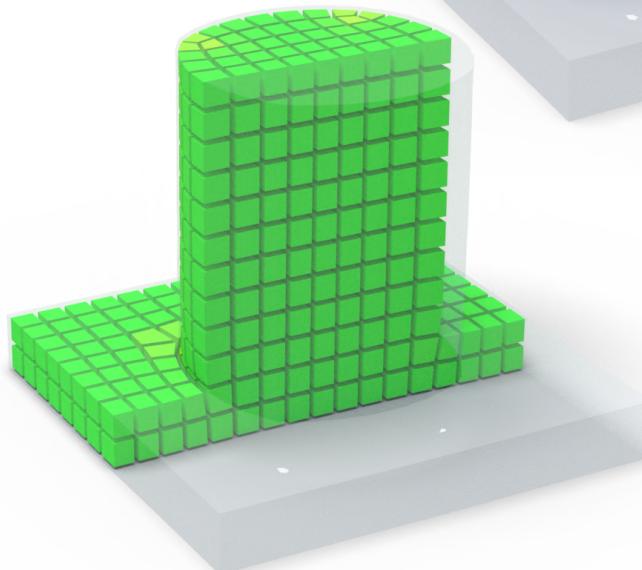
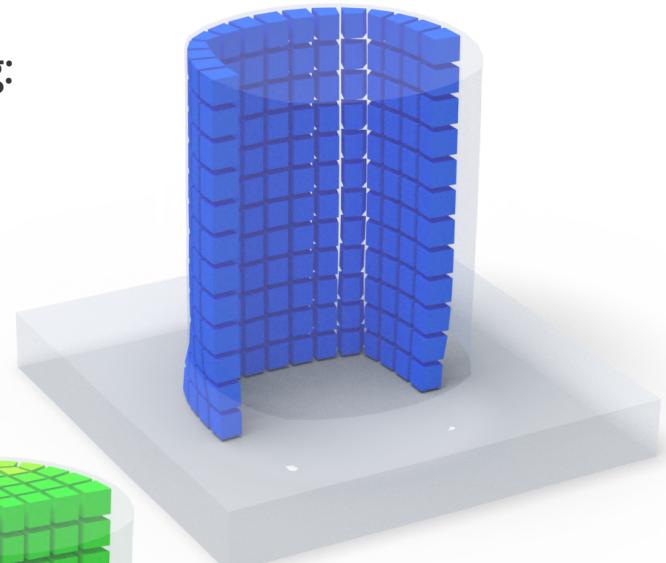
Global padding:

- + elements ✗
- +/- singularities
- quality ✗



Selective padding:

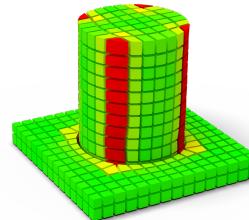
- elements ✓
- +/- singularities
- + quality ✓



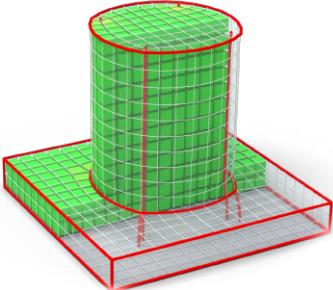
Selective padding pipeline



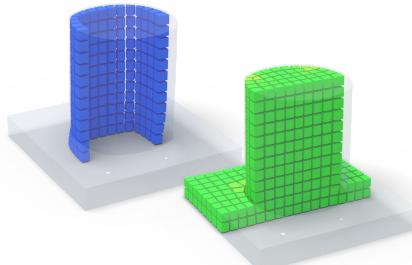
We start from the **model** and its **polycube**



We compute the **hex-mesh** and we **analyse the quality** of the mapping to decide where the padding is required



Now we can **optimize** the hex-mesh with the new degrees of freedom (better quality)



We perform the padding as a **sheet insertion** in the polycube and compute the **new hex-mesh**

We set and solve a **mathematical model** that compute the position of the padding layer

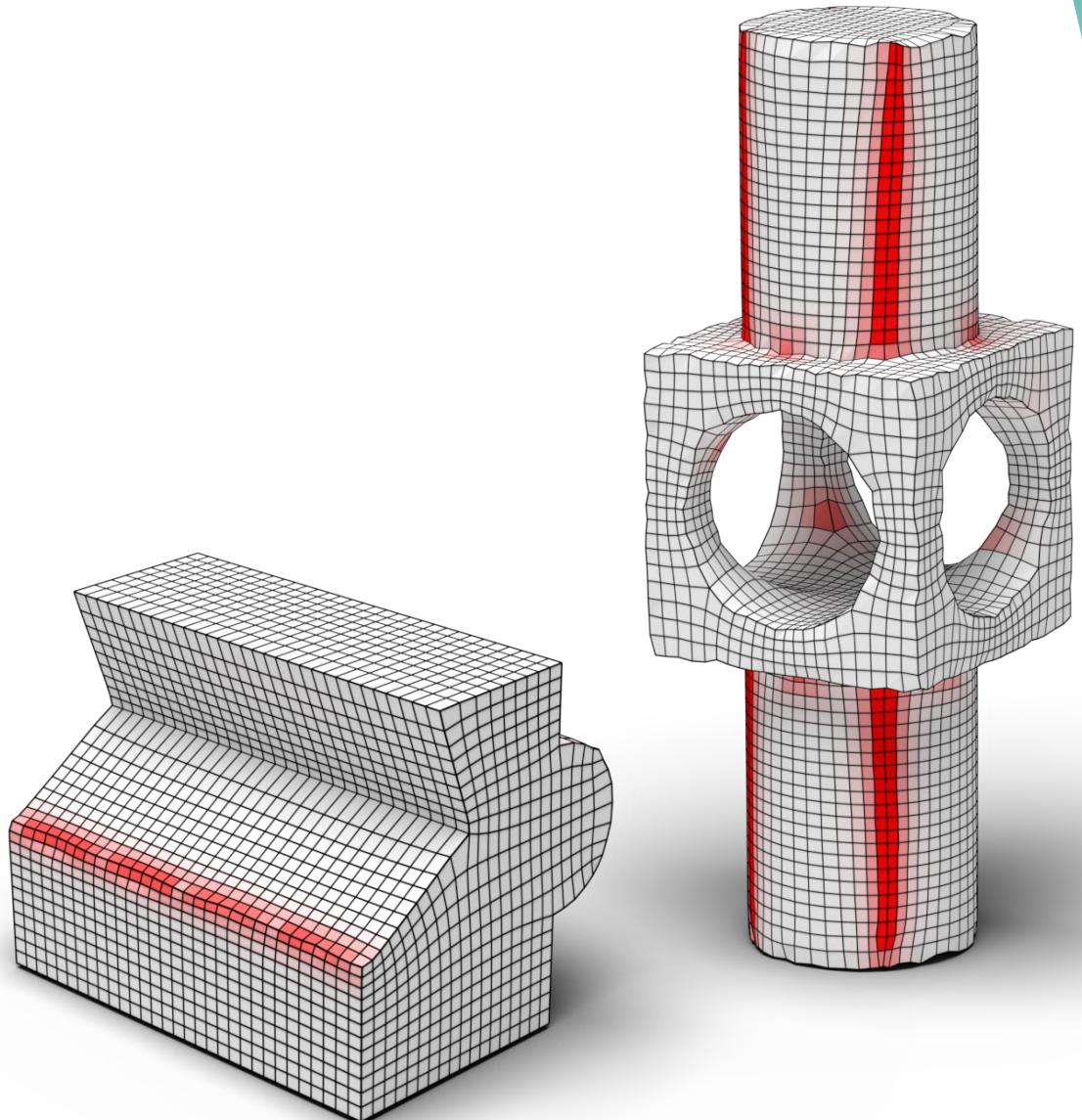
Distortion analysis

The surface of the model is analyzed to find the elements with low quality

We obtain the final set of low quality facets
(HF set)

HF is the set of hard constraints of the mathematical model

NB: Extended analysis pipeline in the paper.



Mathematical model

We want to set a mathematical model as follow:

- **INPUT:** set of faces that require a padding (HF set)
- **OUTPUT:** set of faces to pad
 - Taking care about the number of new hexahedra
 - Taking care about the number of new singularities
 - Taking care about topological consistency

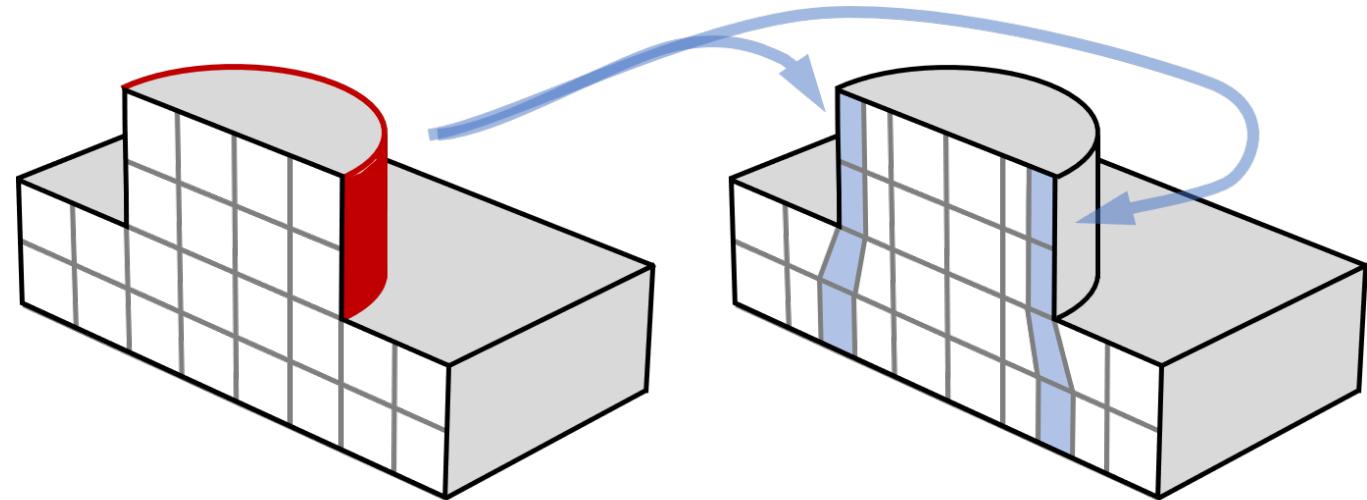
The image shows a green chalkboard with various mathematical equations and diagrams. At the top right, there is a coordinate system with a line equation $(2x+5y)^2 = 2x+12$. Below it, there's a diagram of a circle with radius r and center at $(\frac{r}{2}, \frac{r}{2})$, with a point (x, y) on the circumference. A right triangle is shown with legs of length $\frac{3}{4}x - \frac{1}{2}y$ and $2\frac{3}{4}y$. Other equations include $d = \sqrt{13/2}$, $R = \frac{r\cdot e}{2}$, and $2r = \sqrt{17} + 12^\circ$. There are also some handwritten notes like $278 = u$ and $l = \frac{r\pi a}{18^\circ}$.

Find padding facets

A mathematical model with **binary** and **integer** variables

$$\begin{aligned} \min E = & |E_{padding}| + \lambda \cdot E_{complexity} \\ \text{s.t.} \\ & \text{structural constraints} \end{aligned}$$

$$E_{padding} = \sum_{f_i \in F \setminus HF} x f_i$$



NB: simplified formulas. Extended version of formulas in the paper.

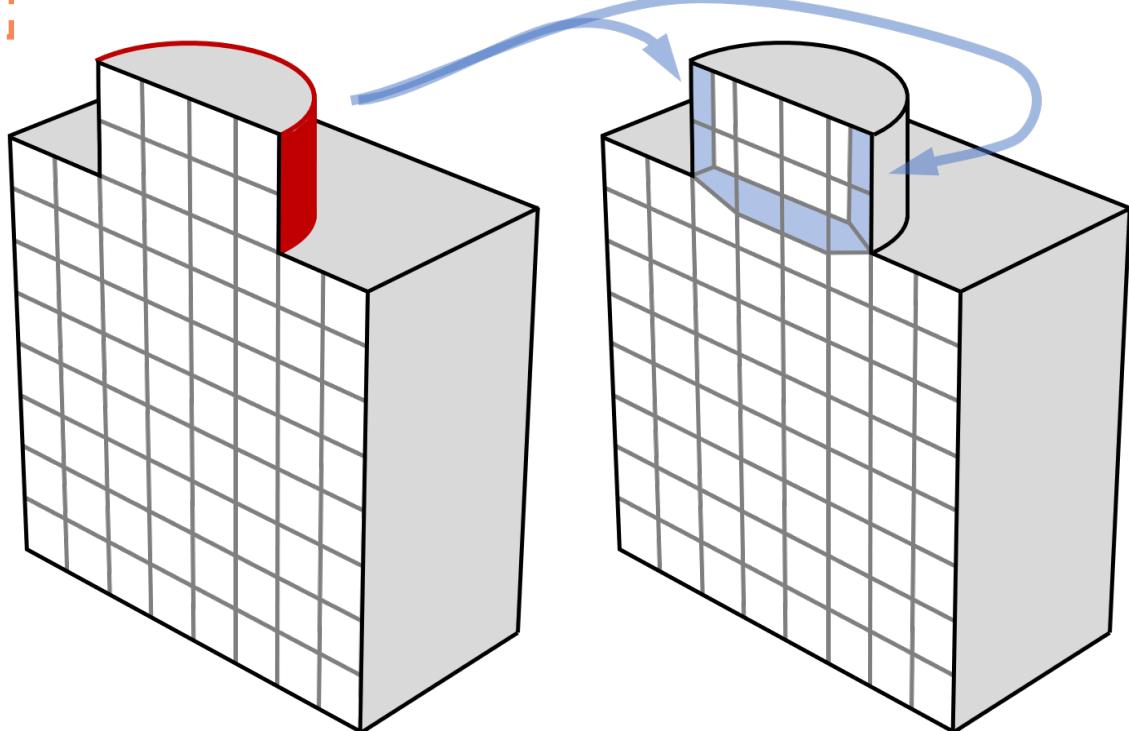


Find padding facets

A mathematical model with **binary** and **integer** variables

$$\begin{aligned} \min E &= E_{padding} + \lambda \cdot E_{complexity} \\ \text{s.t.} \\ &\text{structural constraints} \end{aligned}$$

$$E_{complexity} = \sum_{e_j \in E^*} t e_j + \sum_{v_l \in V^*} t v_l$$



NB: simplified formulas. Extended version of formulas in the paper.

Find padding facets

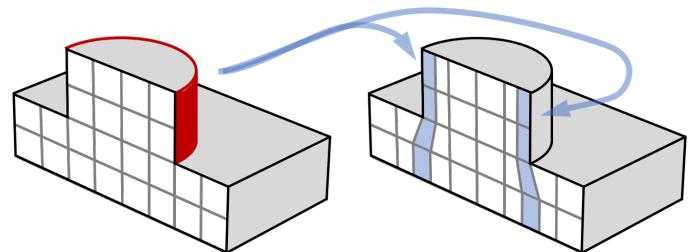
A mathematical model with **binary** and **integer** variables

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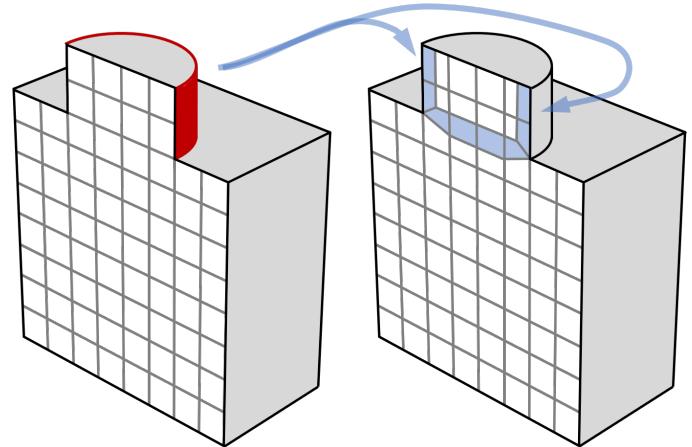
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$$E_{complexity} = \sum_{e_j \in E^*} t e_j + \sum_{v_l \in V^*} t v_l$$

NB: simplified formulas. Extended version of formulas in the paper.



$\lambda = \text{trade-off factor}$



Padding constraints

$$\min E_{padding} = \sum_{f_i \in F \setminus HF} xf_i$$

A binary variable for each facet:

$$xf_i = \begin{cases} 1 & \text{if } f_i \text{ needs padding} \\ 0 & \text{otherwise} \end{cases}$$

$$xf_i = 1 \quad \forall f_i \in HF$$

Hard constraints for facets in HF

$$\sum_{f_i \in F(e_j)} xf_i = 2k_i \quad \forall e_j \in E$$

Constraint to avoid topological inconsistencies during the new layer propagation



Complexity constraints

$$E_{complexity} = \sum_{e_j \in E^*} [te_j] + \sum_{v_l \in V^*} [tv_l]$$

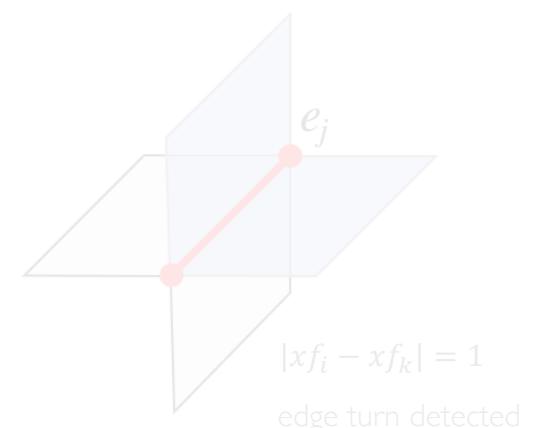
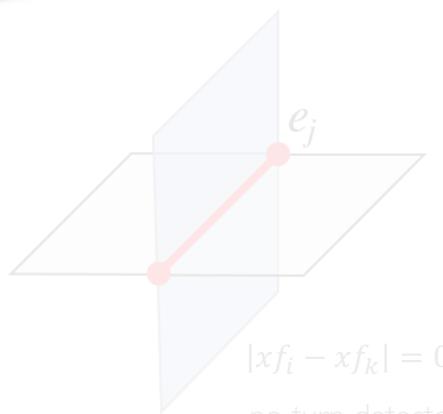
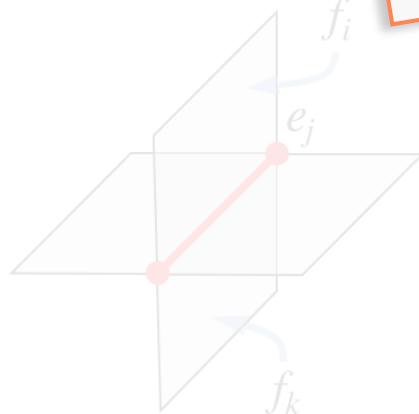
A binary variable for each edge and for each vertex:

$$[x] = \begin{cases} 1 & \text{if there is a turn} \\ 0 & \text{otherwise} \end{cases}$$

Edge turn:

$$te_j = |xf_i - xf_k|$$

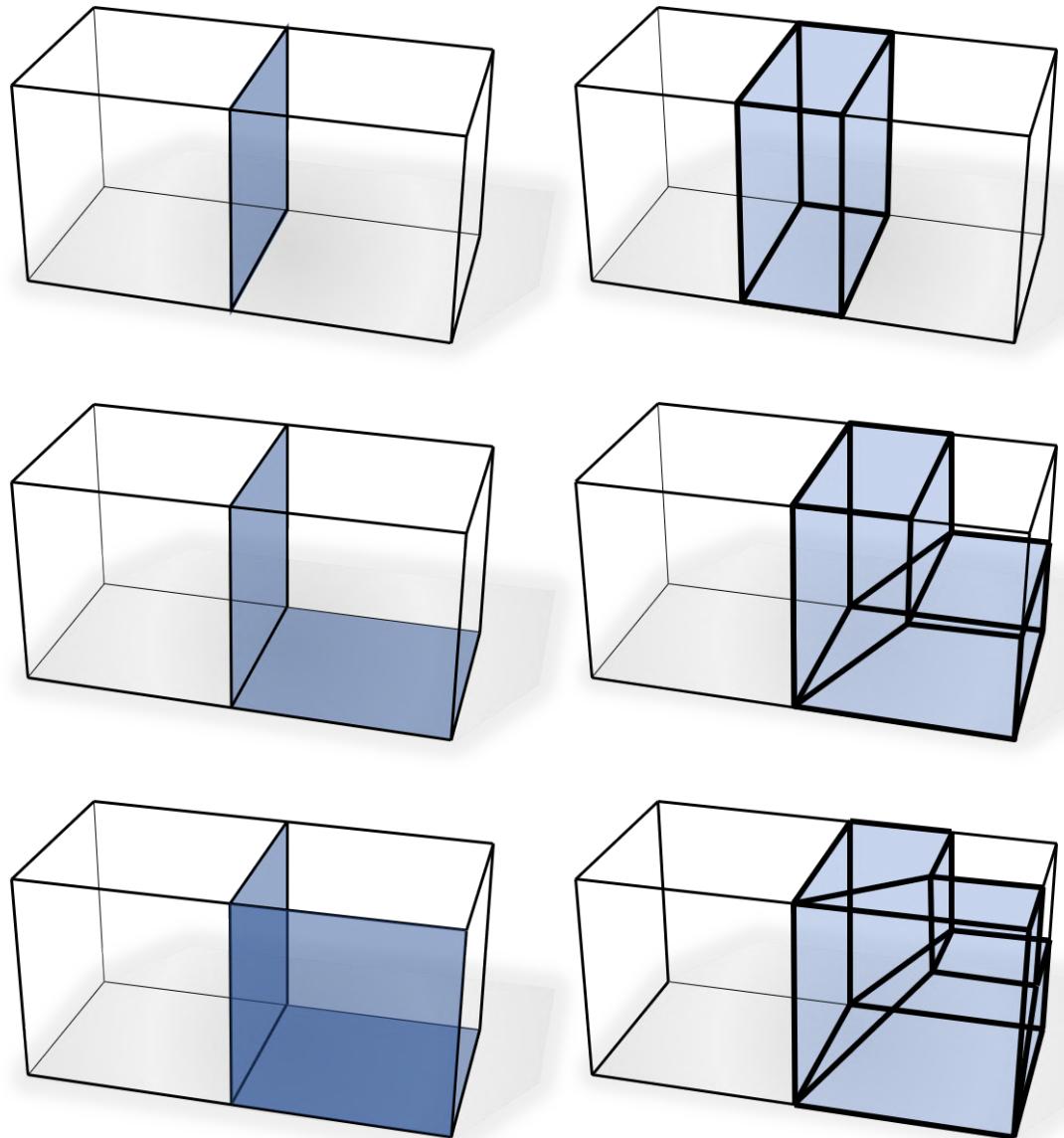
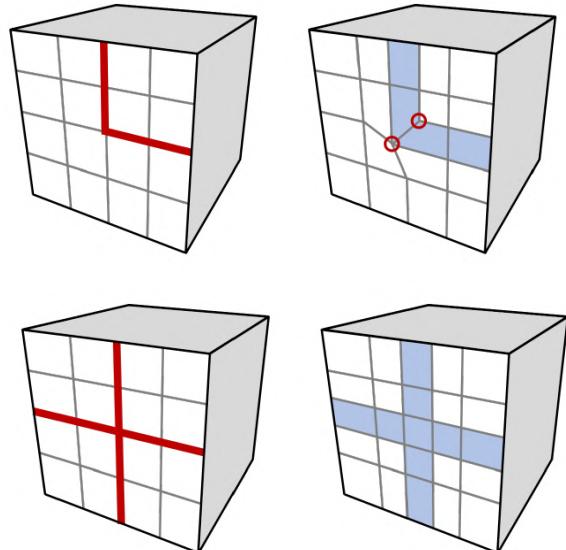
Same reasoning
for vertex turns



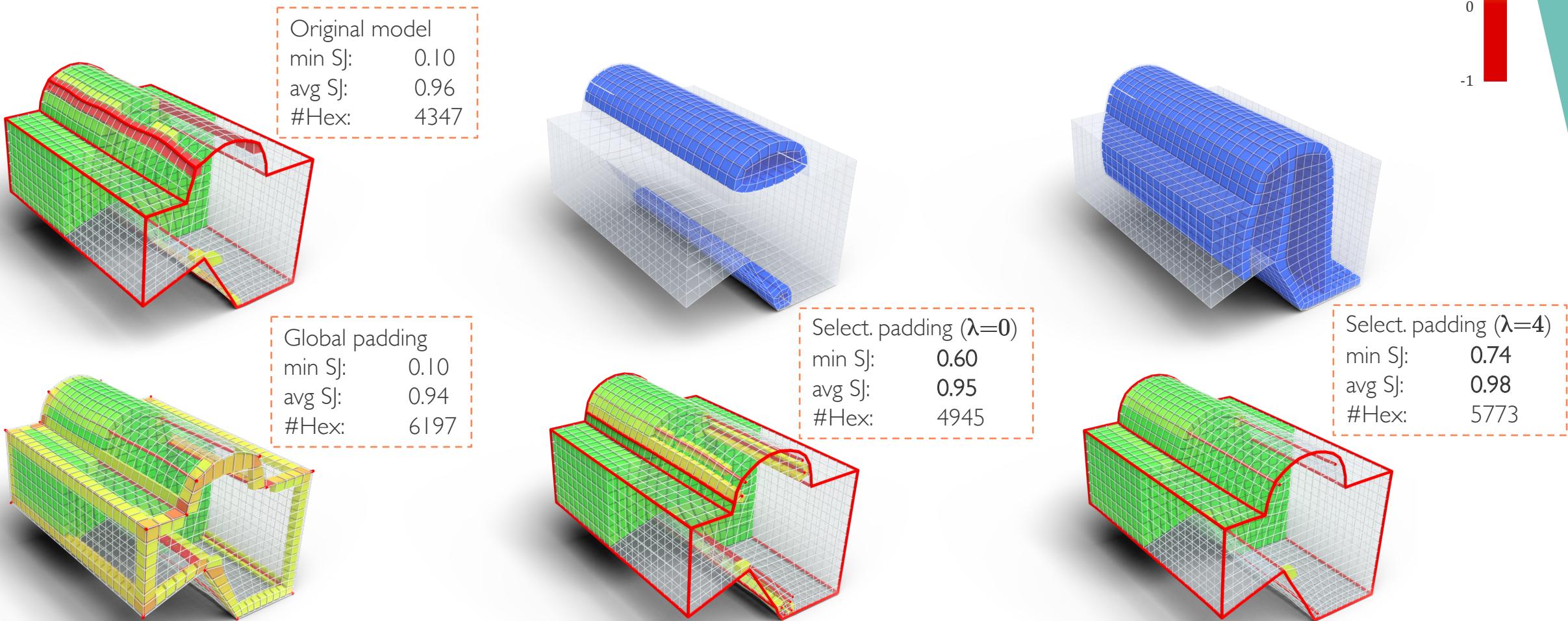
The sheet insertion

Now we know the set of facets
to “pad” to create the padding layer

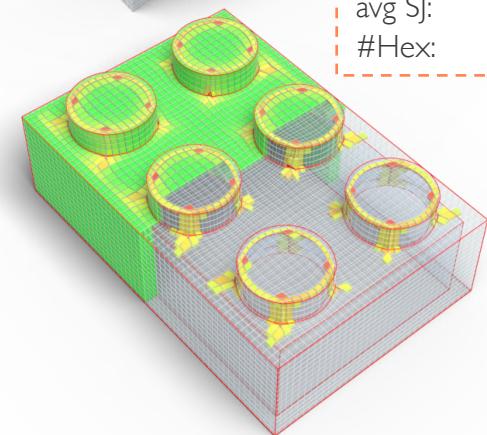
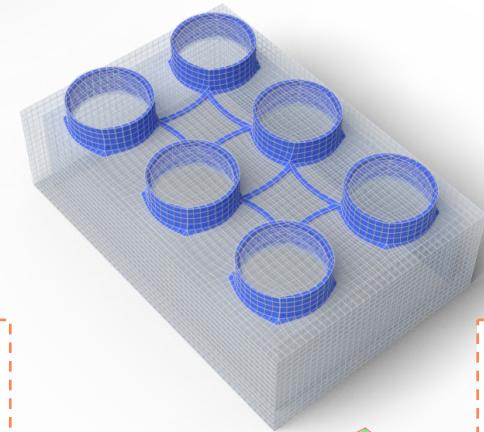
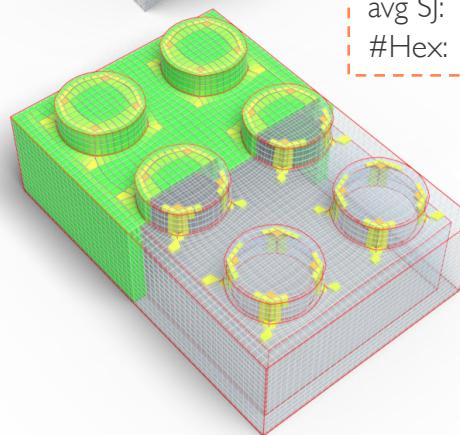
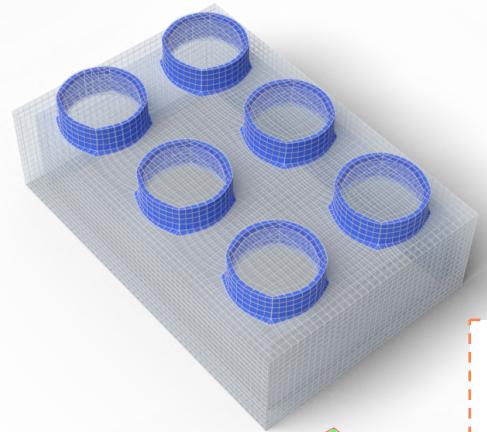
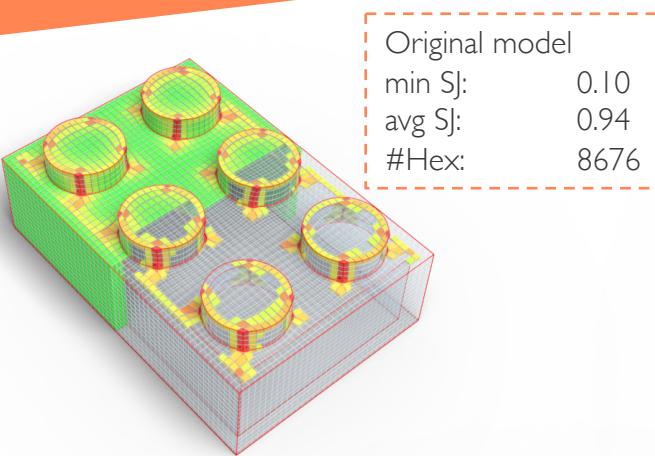
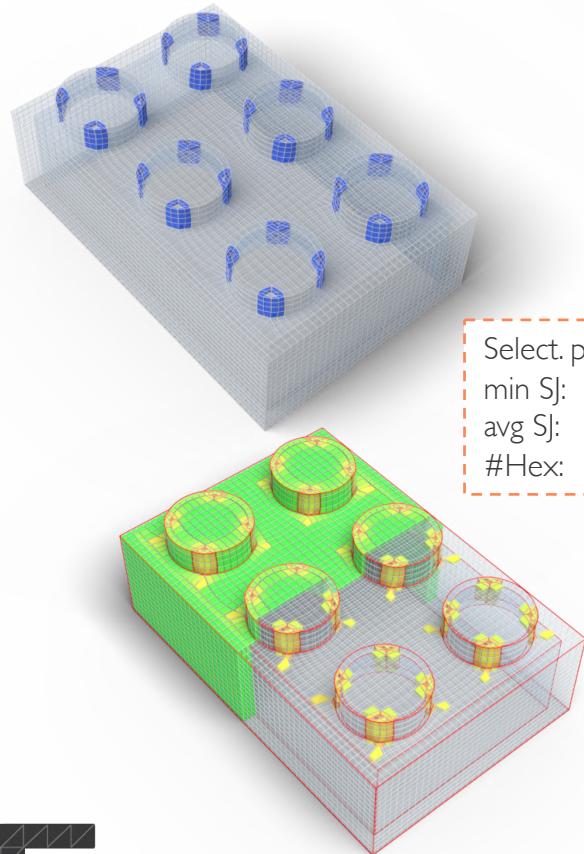
Padding == facet extrusion



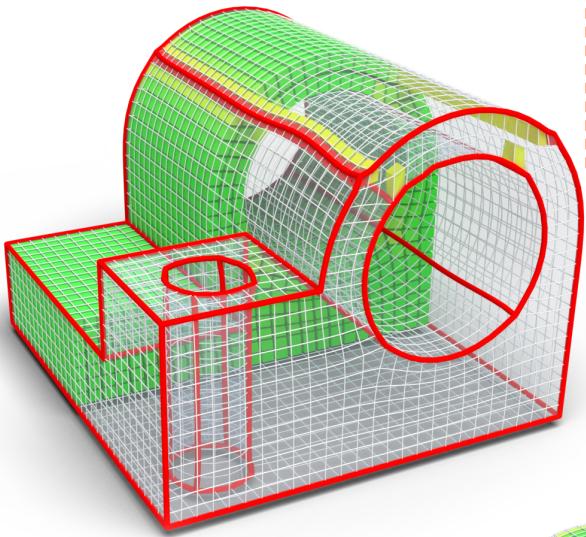
Results



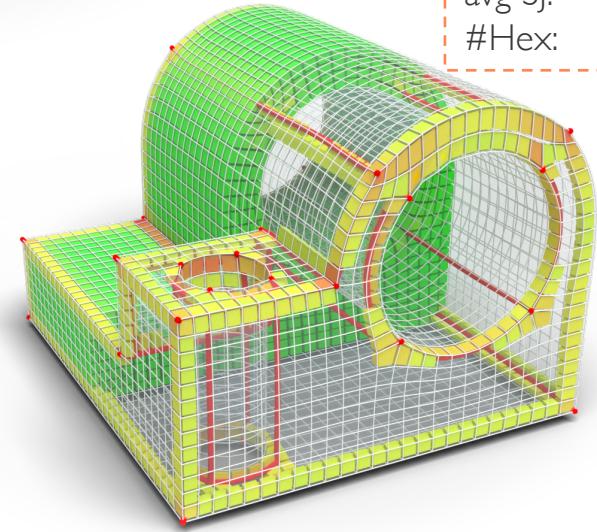
Results



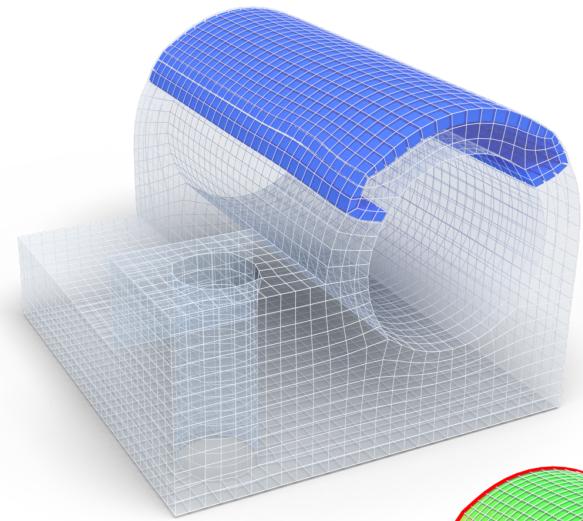
Results



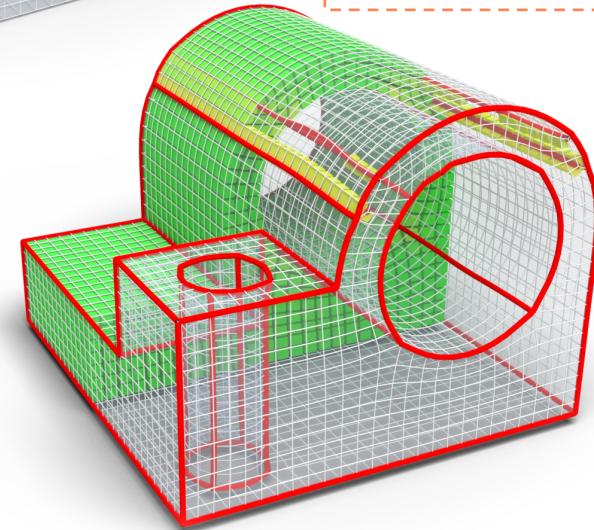
Original model
min SJ: 0.09
avg SJ: 0.97
#Hex: 9032



Global padding
min SJ: 0.16
avg SJ: 0.95
#Hex: 13868



Selective Padding
min SJ: 0.67
avg SJ: 0.97
#Hex: 9872

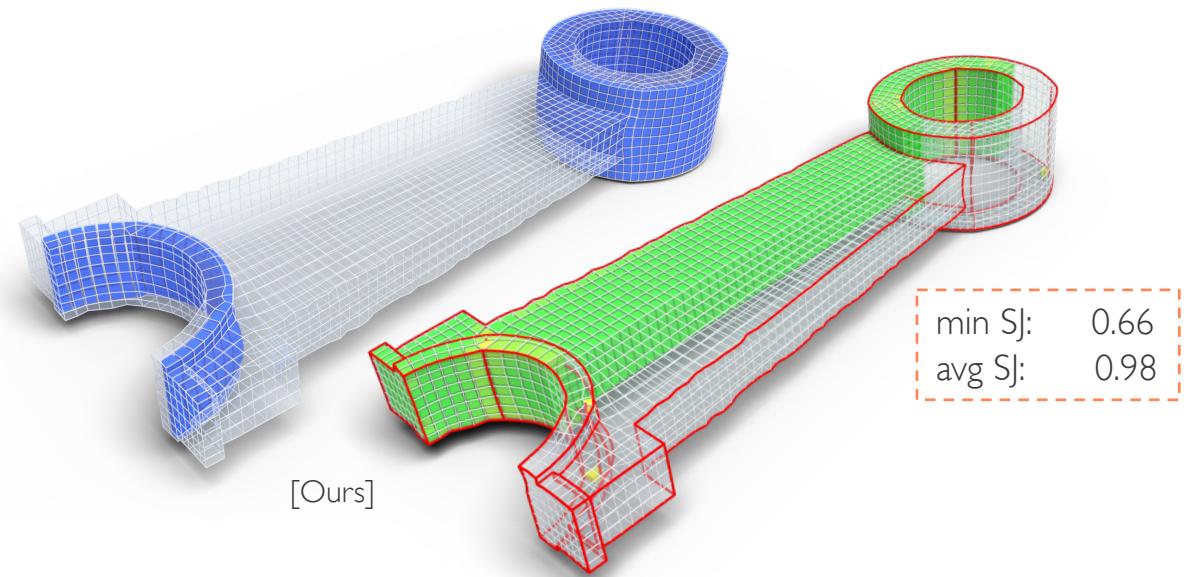
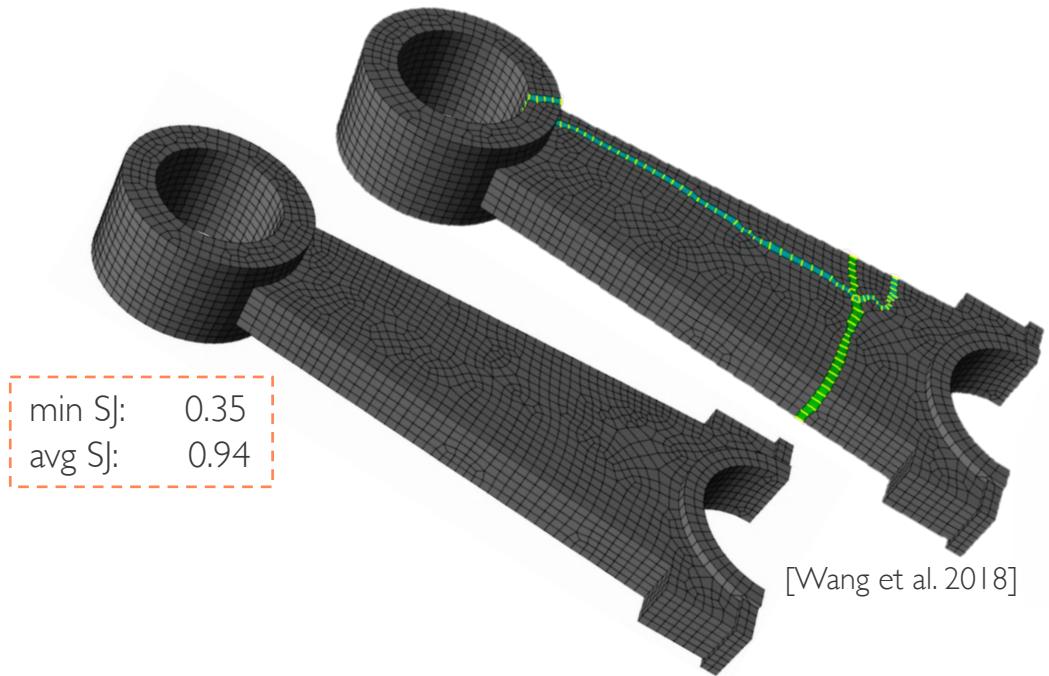


Numbers (timing)

Model	Original model					Global padding					Ours (selective padding)					Time
	#H	#S _v	#S _e	mSJ	aSJ	#H	#S _v	#S _e	mSJ	aSJ	#H	#S _v	#S _e	mSJ	aSJ	
Bearing	7362	64	988	0.07	0.95	12132	128	1052	0.11	0.86	8062	72	1020	0.42	0.97	0.9 s
Block	12408	48	944	0.08	0.93	17896	96	992	0.15	0.95	15216	56	1008	0.69	0.98	2.5 s
Chamfer ($\lambda = 0$)	4347	20	358	0.10	0.96	6197	40	378	0.10	0.94	4945	40	588	0.60	0.95	7.6 s
Chamfer ($\lambda = 4$)	4347	20	358	0.10	0.96	6197	40	378	0.10	0.94	5773	20	366	0.74	0.98	31.6 s
Chamfer (teaser)	10354	20	486	0.02	0.96	13750	40	506	0.13	0.95	12121	28	614	0.61	0.98	40.6 s
Column	940	16	224	0.12	0.94	1790	32	240	0.09	0.91	1276	24	240	0.81	0.97	0.2 s
Double hinge (NH)	3120	24	424	0.03	0.94	5342	48	448	0.31	0.89	3770	40	536	0.65	0.95	3.1 s
Double hinge (WH)	3120	24	424	0.03	0.94	13868	64	712	0.16	0.95	4550	40	536	0.63	0.95	4.2 s
Gear	6816	72	796	0.03	0.96	10136	144	868	0.03	0.93	8640	72	812	0.70	0.98	8.2 s
Joint	9032	32	680	0.09	0.97	13868	64	712	0.16	0.95	9872	40	804	0.67	0.97	5.3 s
Lego ($\lambda = 0$)											9372	520	2596	0.39	0.94	2.3 s
Lego ($\lambda = 2$)	8676	112	1828	0.10	0.94	18810	224	1940	0.13	0.77	9876	160	1876	0.54	0.97	3.9 s
Lego ($\lambda = 4$)											9940	176	1900	0.31	0.95	8.9 s
Wrench	1576	32	472	0.06	0.95	3576	64	508	0.14	0.89	1796	40	492	0.71	0.97	2.7 s
Test 1	4272	72	832	0.07	0.95	7688	144	904	0.03	0.90	5080	72	852	0.66	0.98	4.06 s
Test 2	4752	40	520	0.06	0.92	7026	80	560	0.10	0.87	6672	56	556	0.73	0.96	20.7 s
Test 4	9779	56	812	0.14	0.97	14391	112	868	0.17	0.95	11730	56	824	0.68	0.99	2.6 s
Test 5	50830	84	2266	0.10	0.95	73666	168	2350	0.19	0.94	53686	84	2370	0.41	0.98	49.8 s



Comparison



- Comparable results (usually better)
- More regular inner structure (and less singularities)
- We can focus the analysis only on the model surface



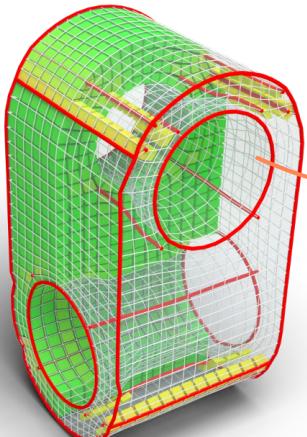
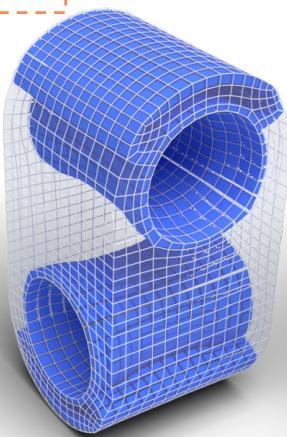
Limitations

- How to choose the λ parameter?
- Padding “holes”

*to pad, or not to pad?
that's the question*

Selec. Padding (holes)

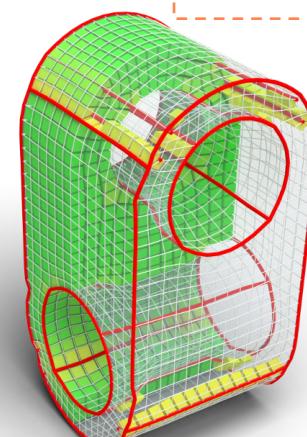
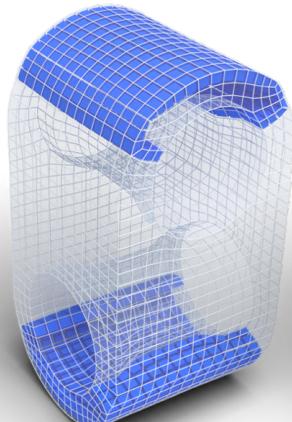
min SJ:	0.63
avg SJ:	0.95
#Hex:	4550



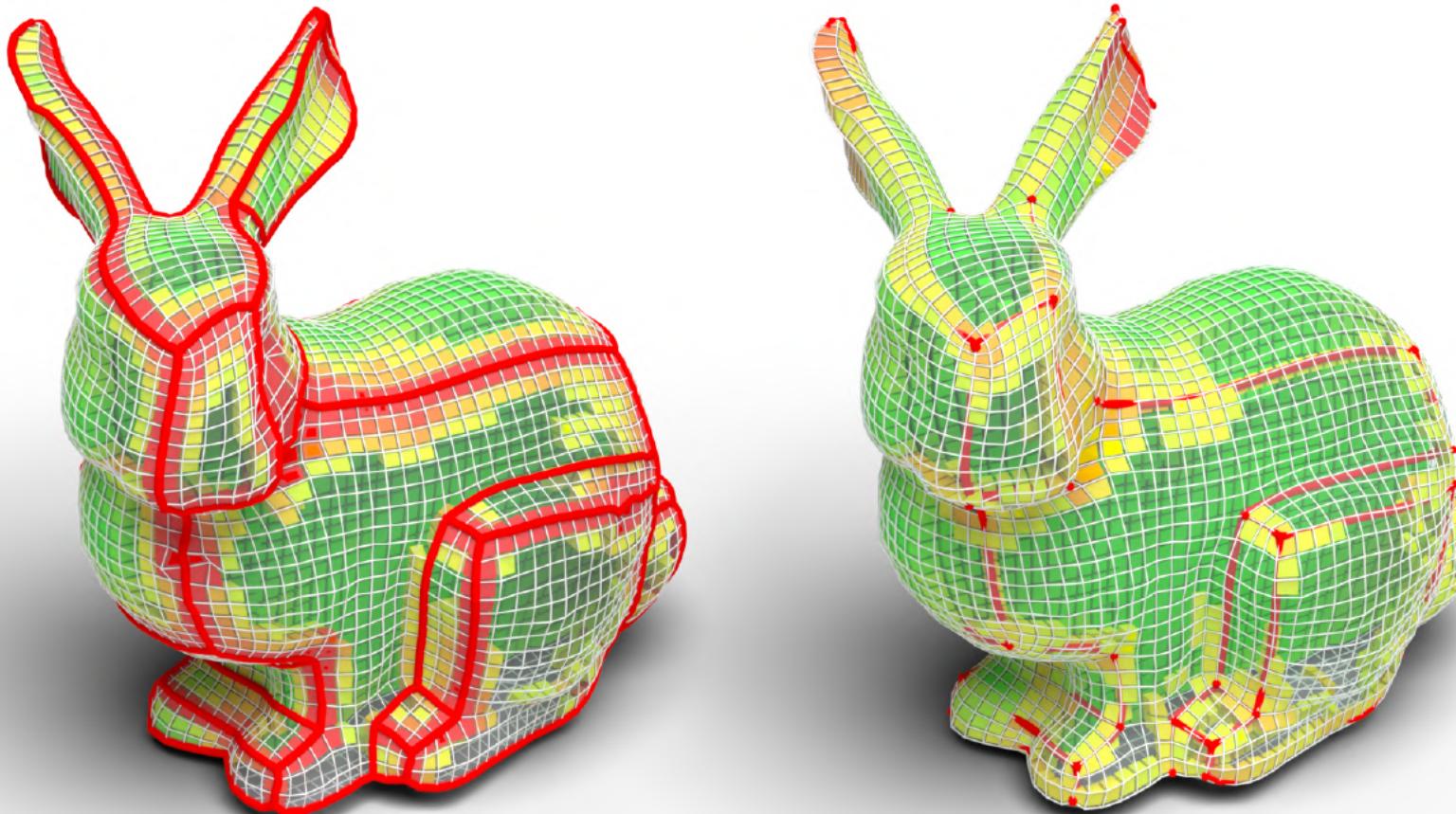
*object shape
object final use
object resolution*

Selec. Padding (NO holes)

min SJ:	0.65
avg SJ:	0.95
#Hex:	3770



Mechanical parts vs Free-form shapes



Conclusion and Future work

Algorithm to perform **Selective Padding** in polycube-based hex-meshes:

- (only) surface quality analysis
- automatic insertion of new hexahedral layers
- singularity and number of elements trade-off

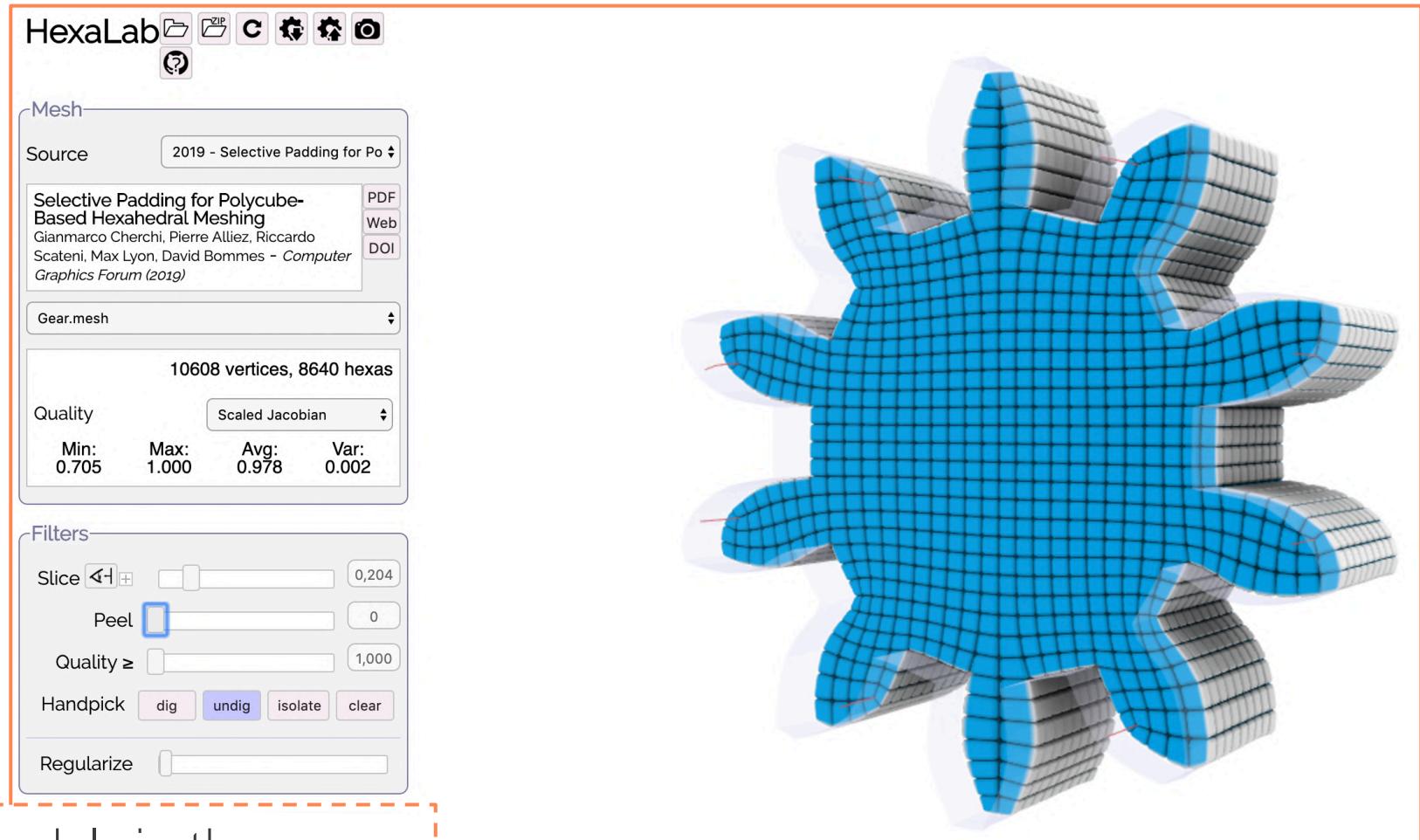


In the future:

- automatic λ selection
- padding vs inverse padding (remove layers)
- more general polycube-based meshes



Get models



All models in the paper
in www.hexalab.net

Thanks!

