



**SIGGRAPH 2022**  
VANCOUVER+ 8-11 AUG

THE PREMIER CONFERENCE & EXHIBITION ON  
COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES

# OPTIMAL DUAL SCHEMES FOR ADAPTIVE GRID BASED HEXMESHING

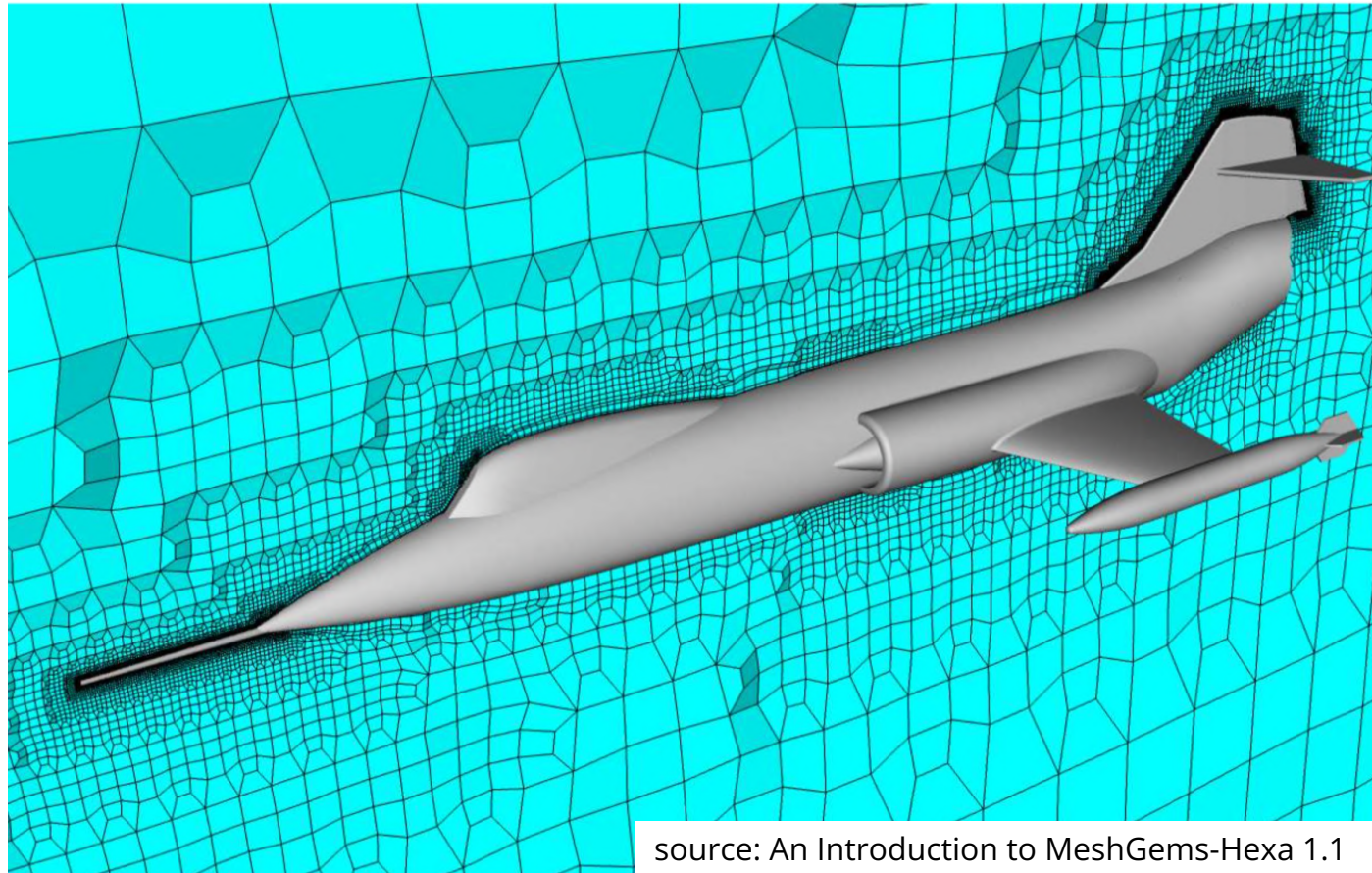
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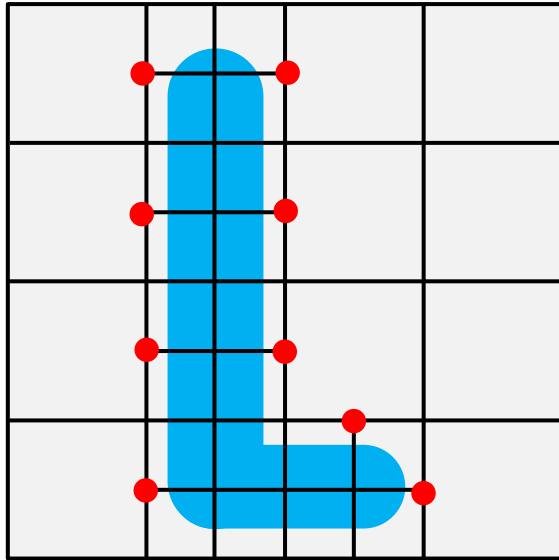
# Grid-based hexahedral meshing

- ▶ The only existing automatic method for "*black box*" hexahedral meshing
- ▶ **Theoretical breakthrough:** dual methods [Maréchal, 2009]

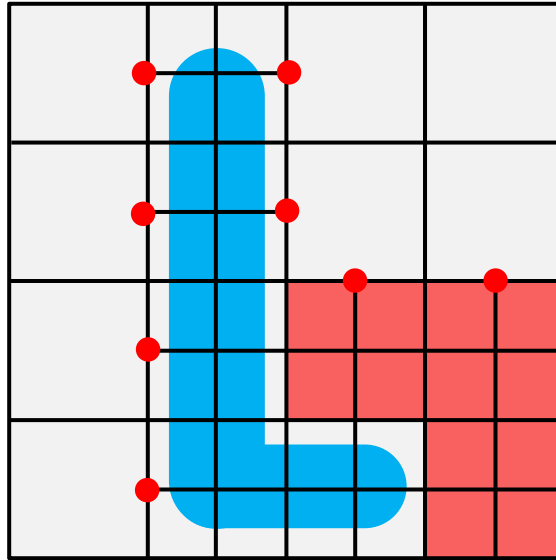


source: An Introduction to MeshGems-Hexa 1.1

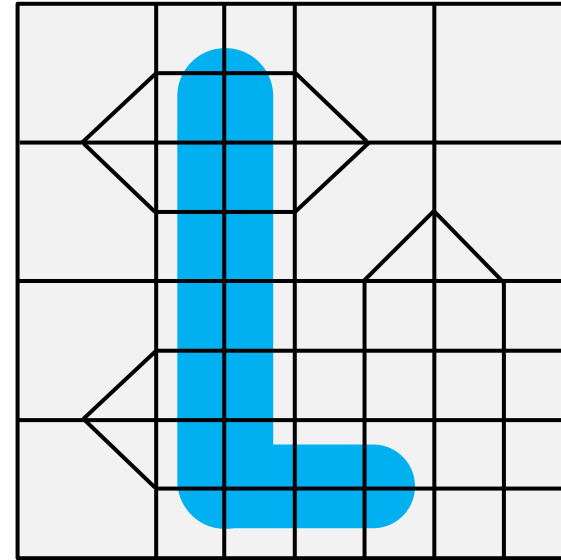
# Basics of dual grid-based hexmeshing



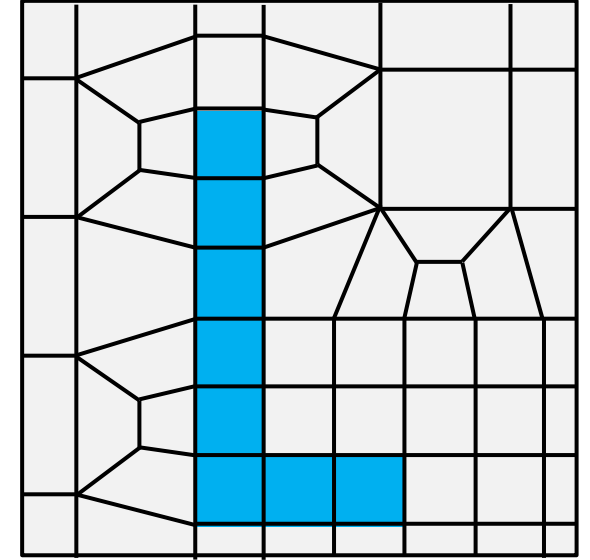
Initialize an adaptive grid



Refine it more to make it hexable



Suppress hanging nodes



Dualize to obtain a hexmesh



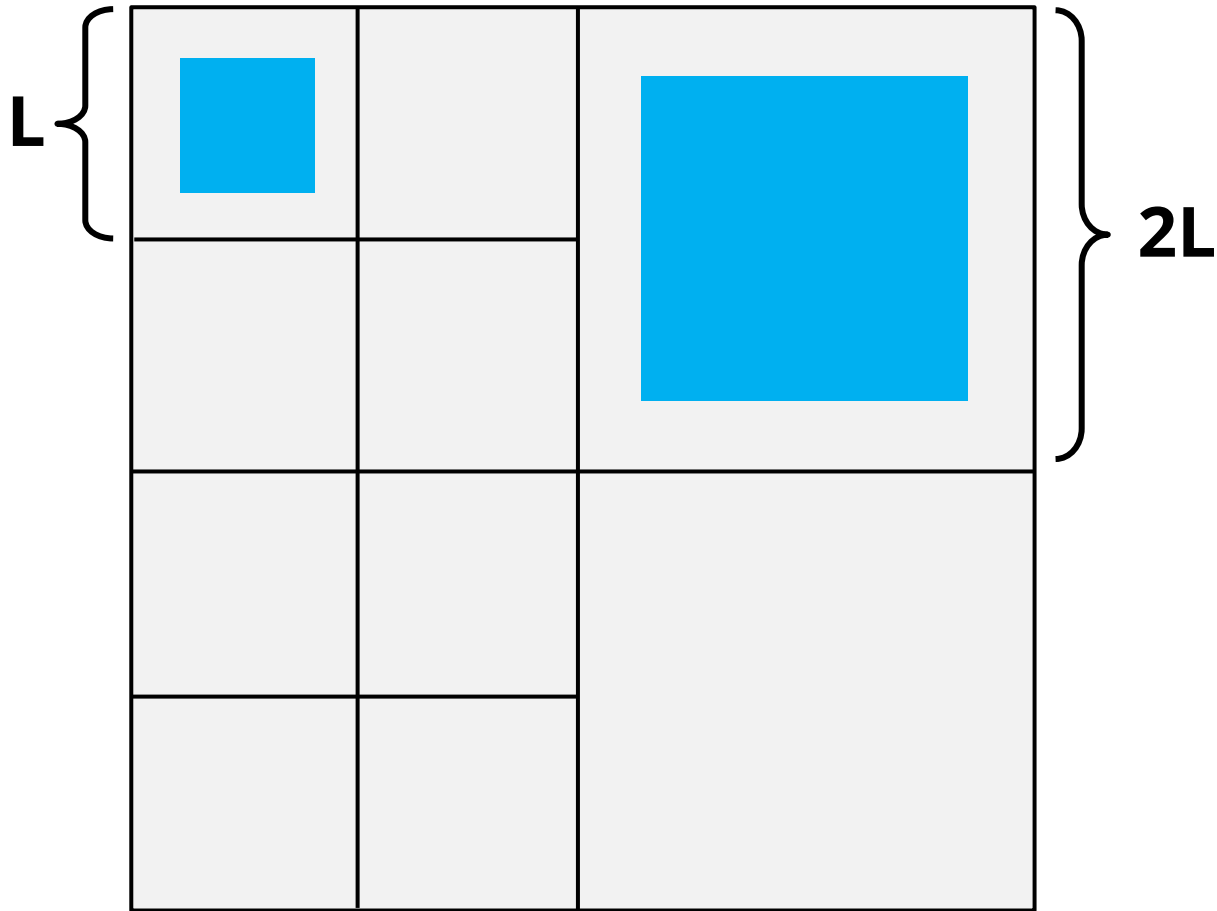
MAJOR  
BOTTLENECK  
(~10x)



DARK  
MAGIC

# Requirements

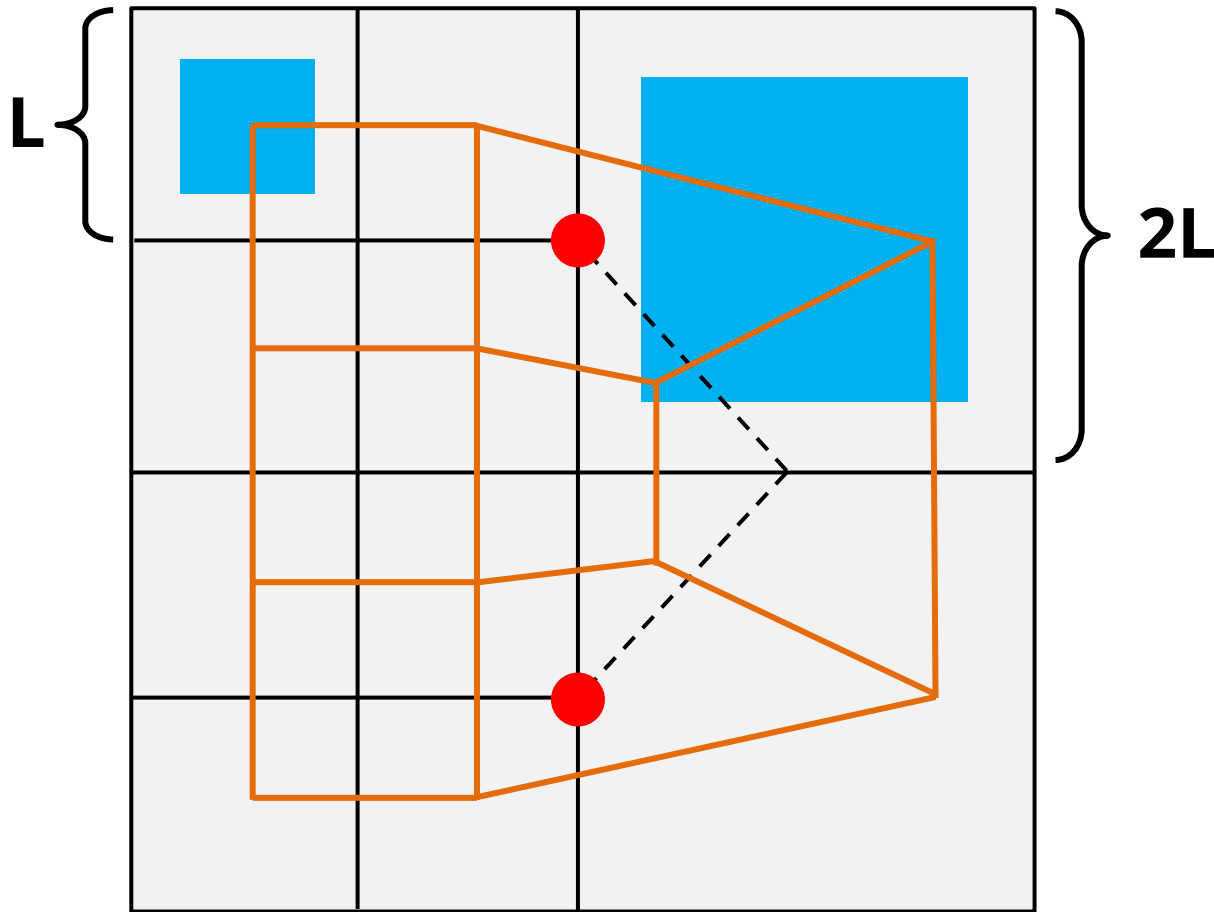
Hanging nodes can be suppressed only if two topological conditions are met



**BALANCING:** size mismatch for neighbors must be at most one

# Requirements

Hanging nodes can be suppressed only if two topological conditions are met



**BALANCING:** size mismatch for neighbors must be at most one

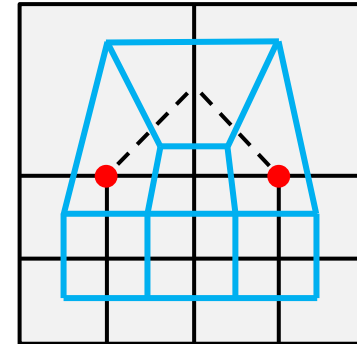
**PAIRING:** number of hanging nodes must be even along all sides

# Transition schemes

Given a 3D adaptive grid that fulfills balancing and pairing, how to regularize vertex valence?

► In 2D:

- each primal vertex with valence 4 yields a dual quad
- one transition scheme handles all possible cases



► In 3D:

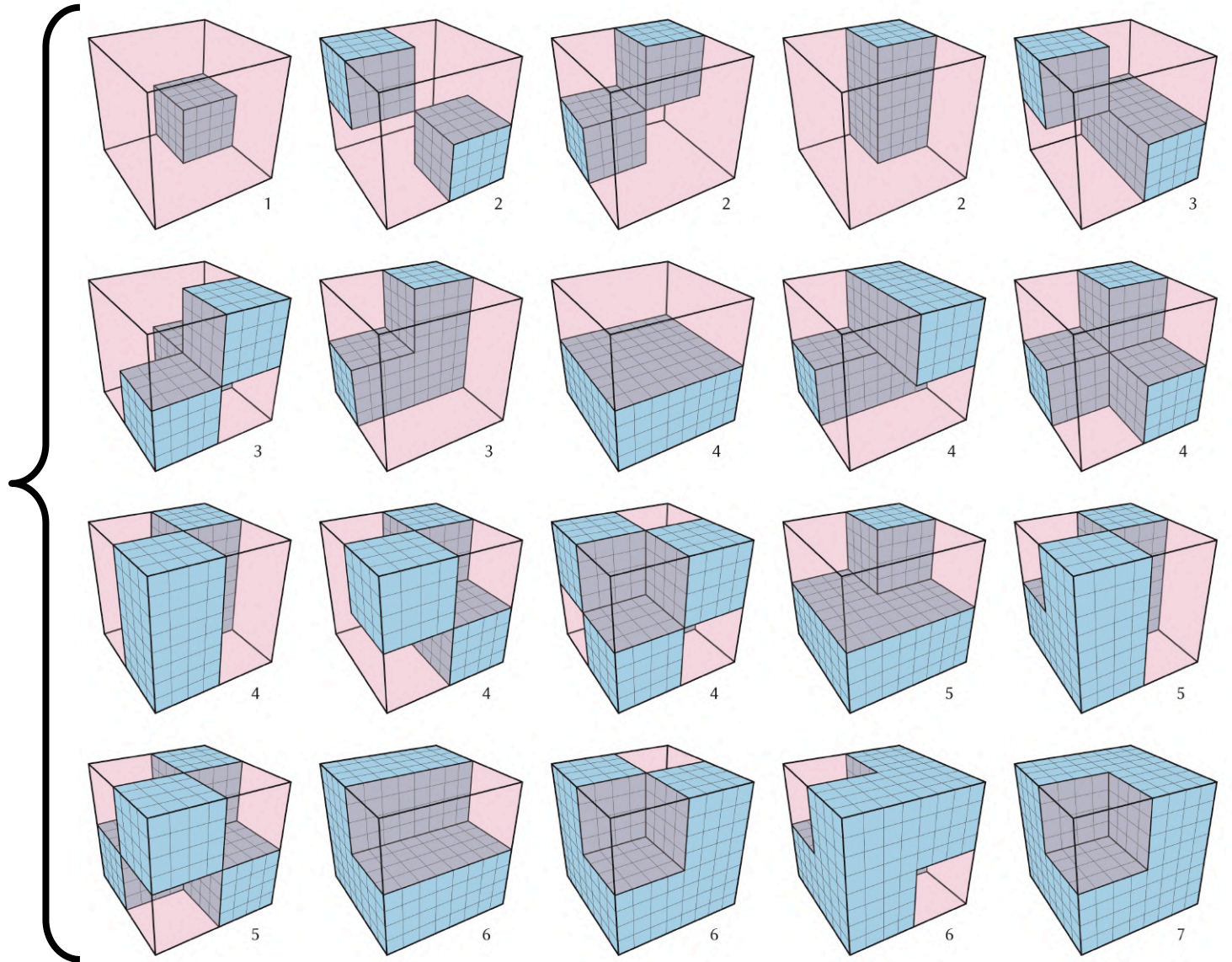
- each primal vertex with valence 6 yields a dual cell with 6 faces
- each primal edge with valence 4 yields a dual quadrilateral face
- **more transition schemes are necessary**

ALL HEX  
CONDITIONS

# Combinatorics

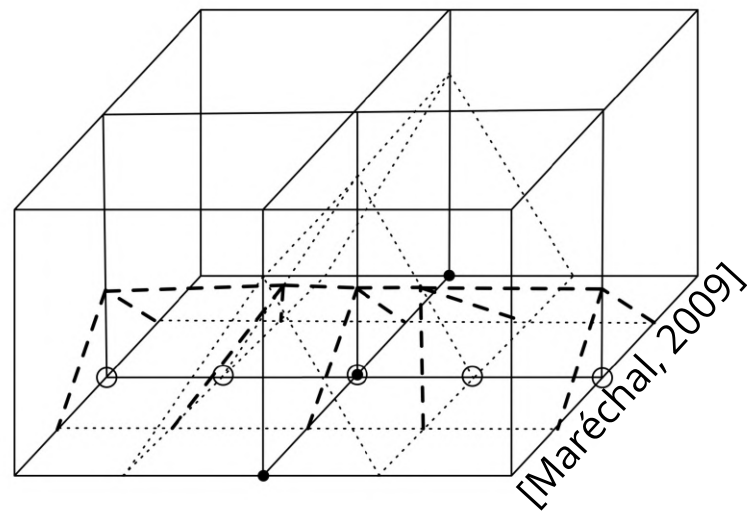
20  
POSSIBLE  
TRANSITIONS\*

\*assuming a BALANCED grid

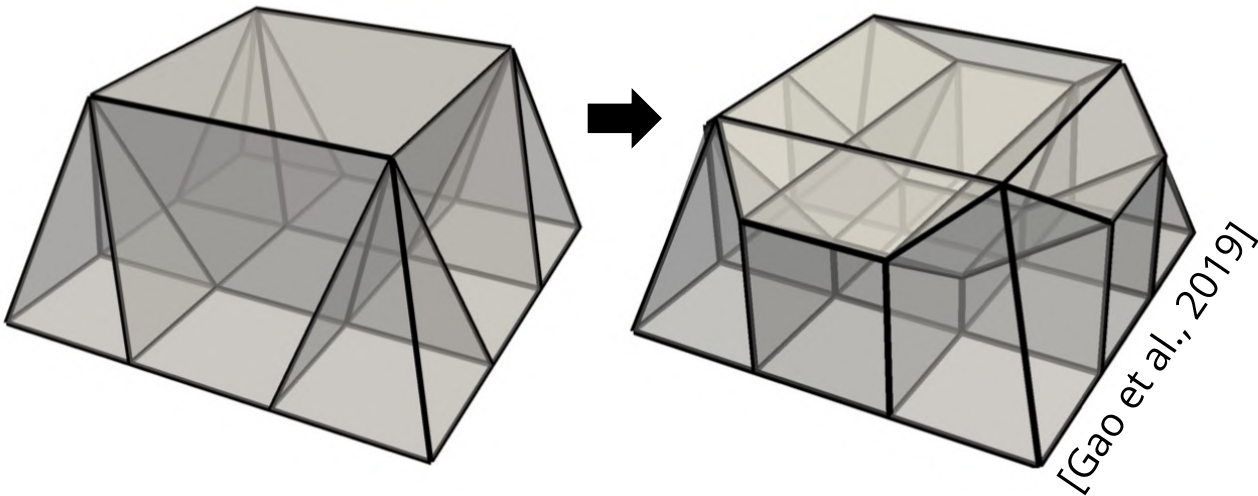


# State of the art

## REGULARIZE VERTEX/EDGE VALENCES VIA CELL CUTTING



## DUALIZE FIRST, THEN SUBSTITUTE CLUSTERS OF HYBRID CELLS WITH HEXA



DO NOT NATIVELY COVER ALL 20 POSSIBLE TRANSITIONS

FLAT (4x4 to 2x2)

CONVEX EDGE

CONCAVE EDGE

CONCAVE CORNER

FLAT (4x4 to 2x2)

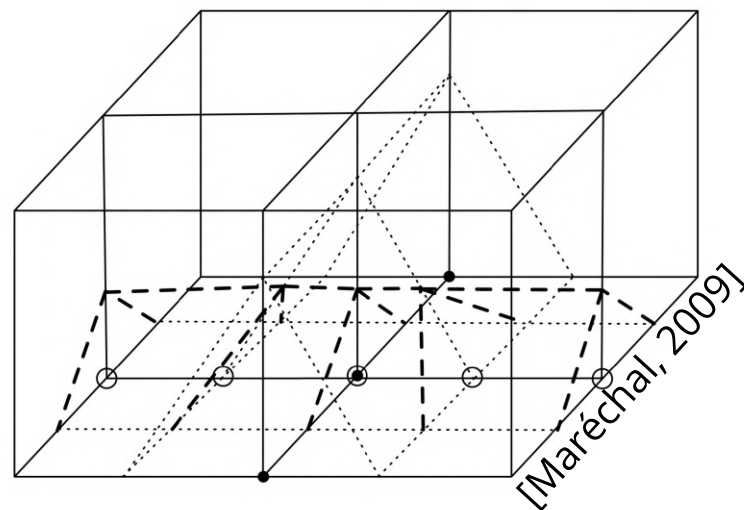
CONVEX EDGE

CONCAVE EDGE

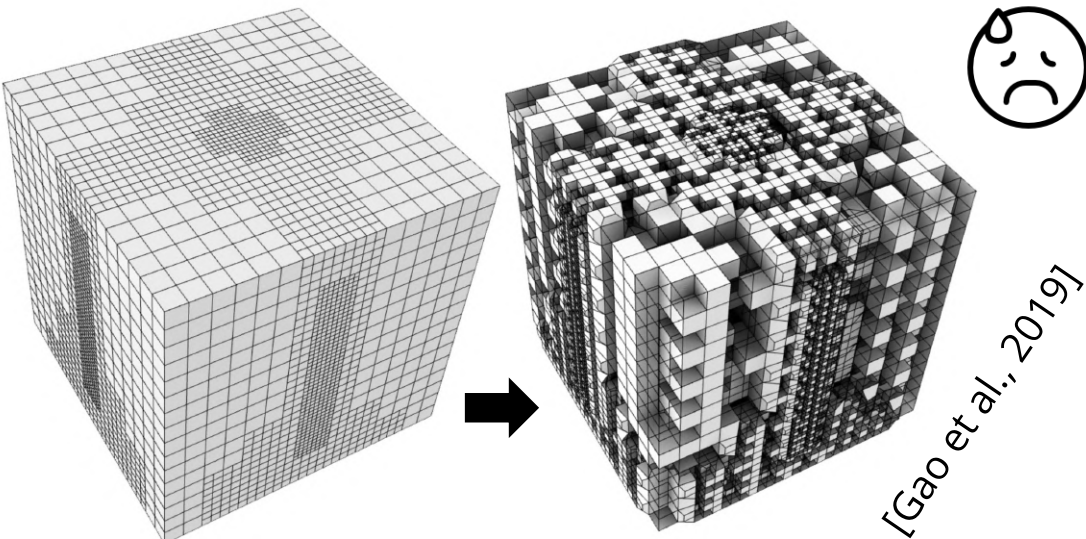
CONCAVE CORNER

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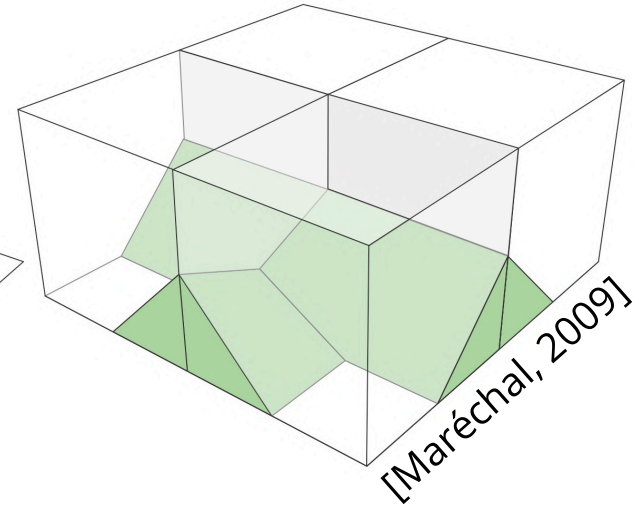
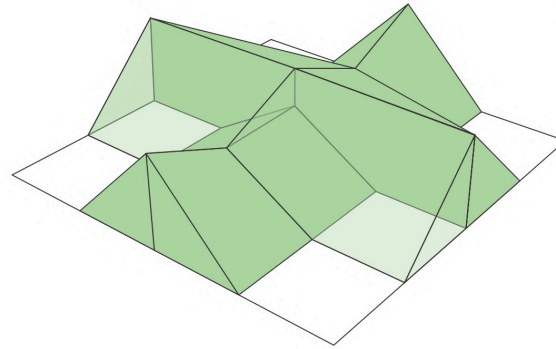
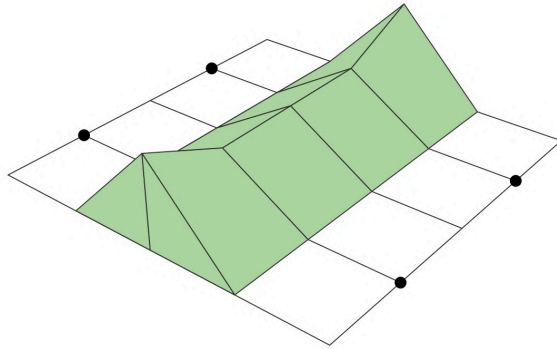
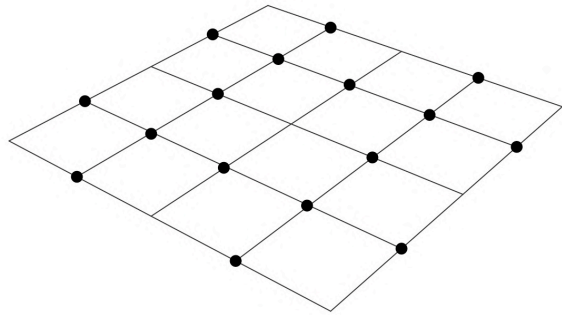
CONCAVE EDGE

CONCAVE CORNER

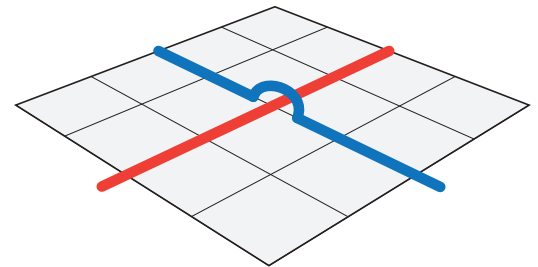
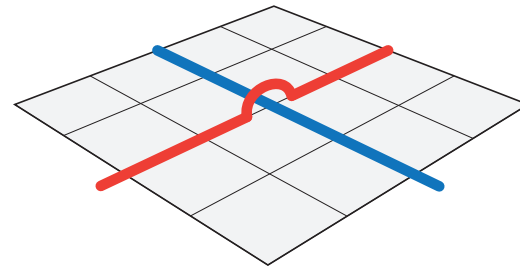
HEURISTIC CLUSTERING MAY FAIL

# A dive into transitions

The idea is to suppress hanging nodes by ***chains of prismatic elements*** around clusters of refined cells

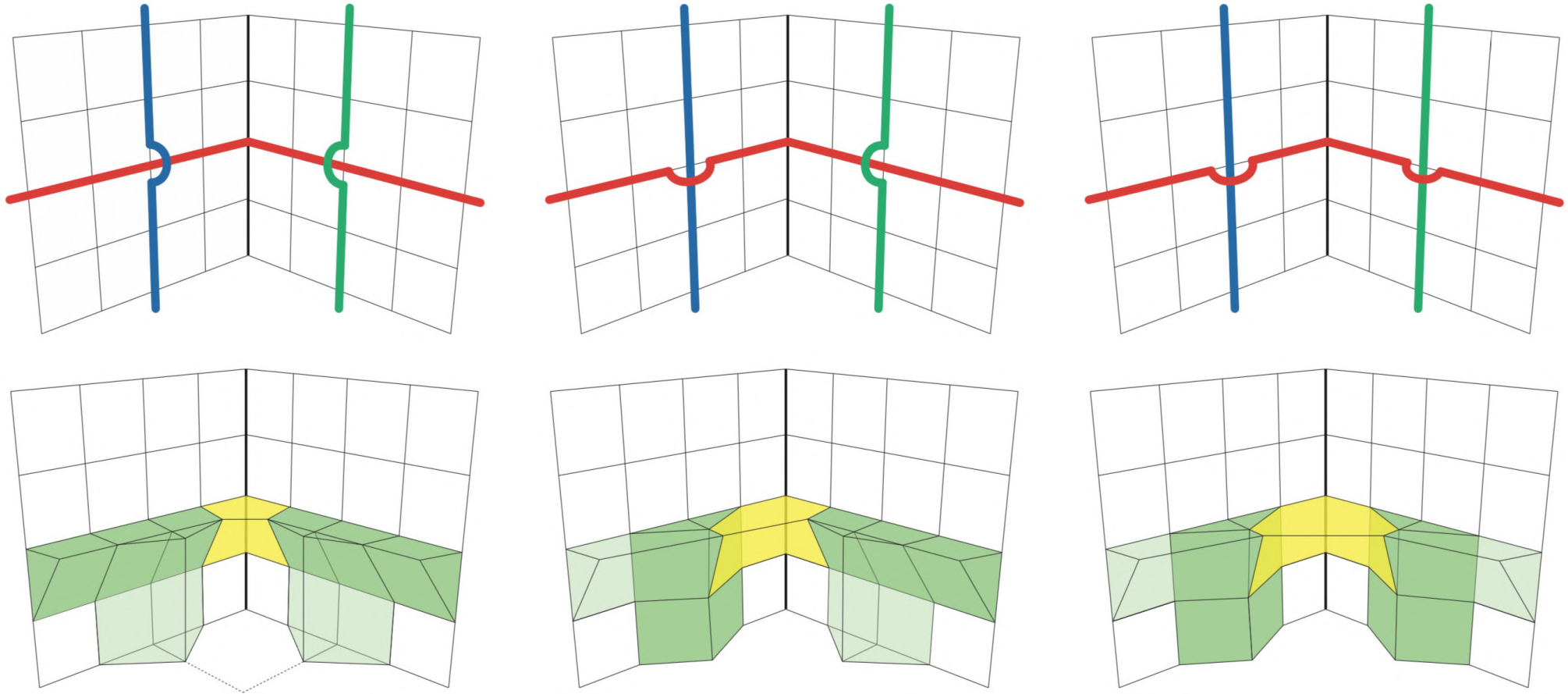


- schemes are internally ***asymmetric***
  - which chain goes above and which below?
  - **ambiguities** in the implementation
  - unwanted effects on the **singular structure**



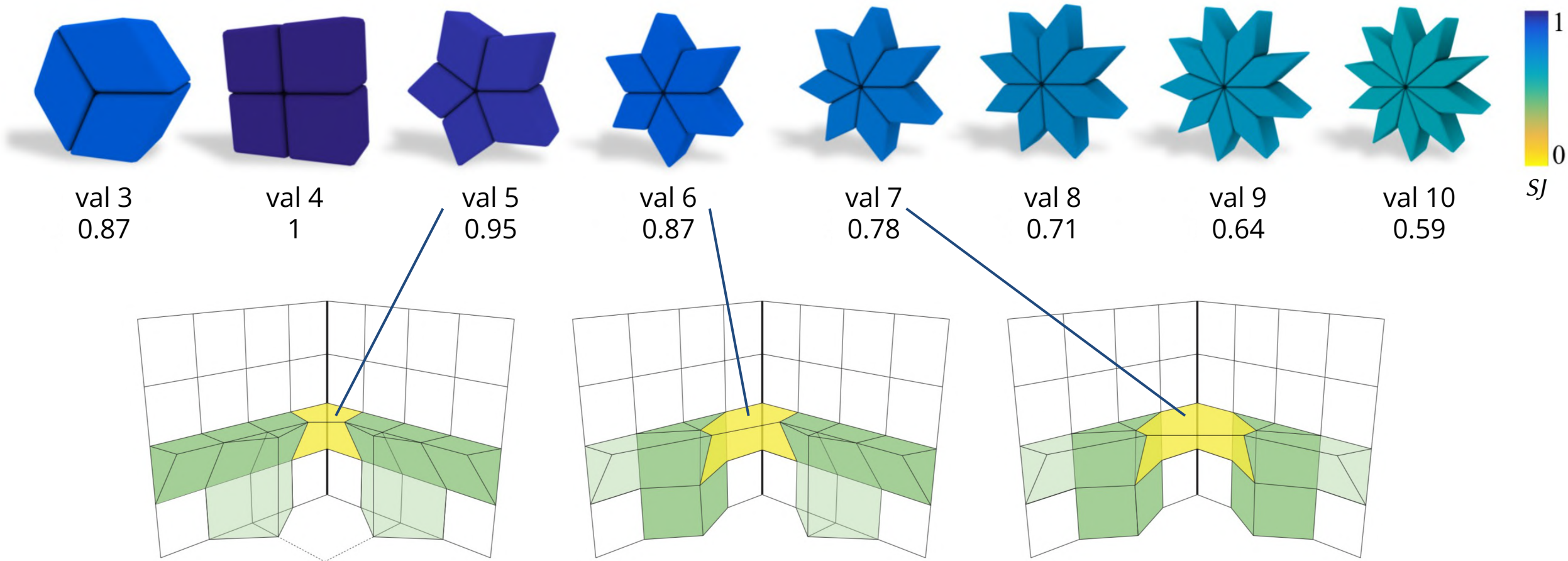
# Concave edges

There are three possible ways to handle a concave transition



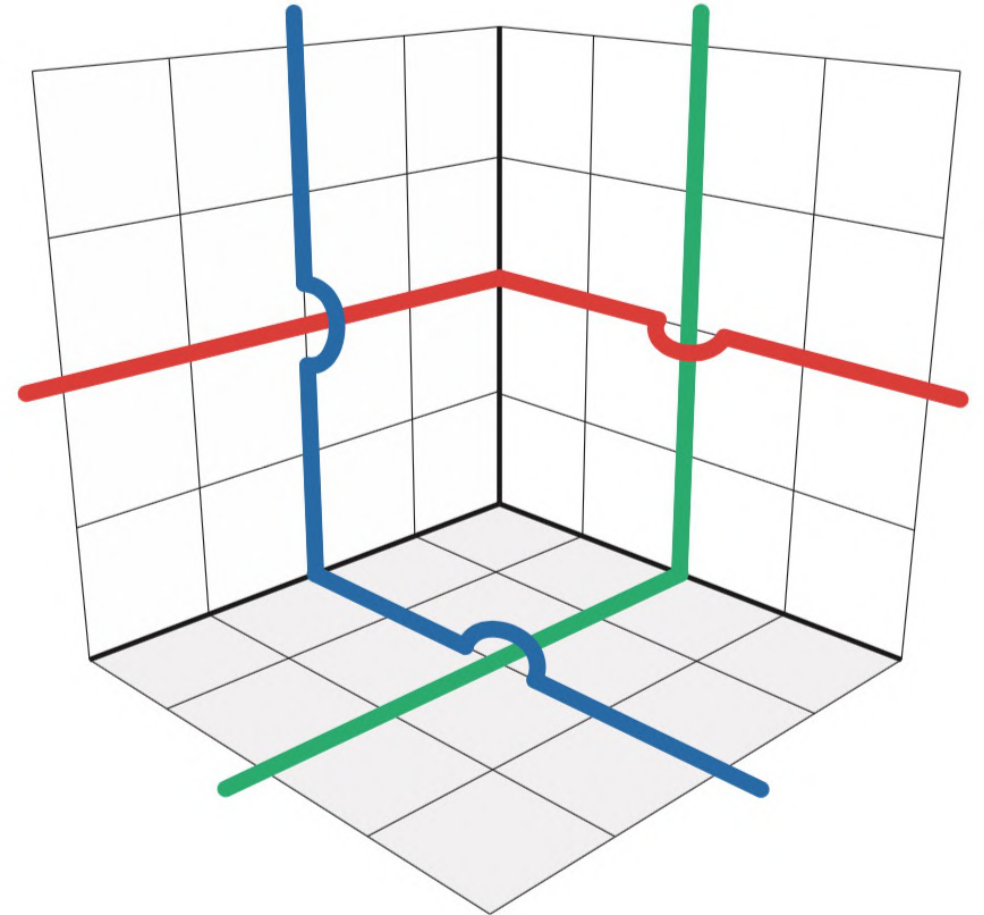
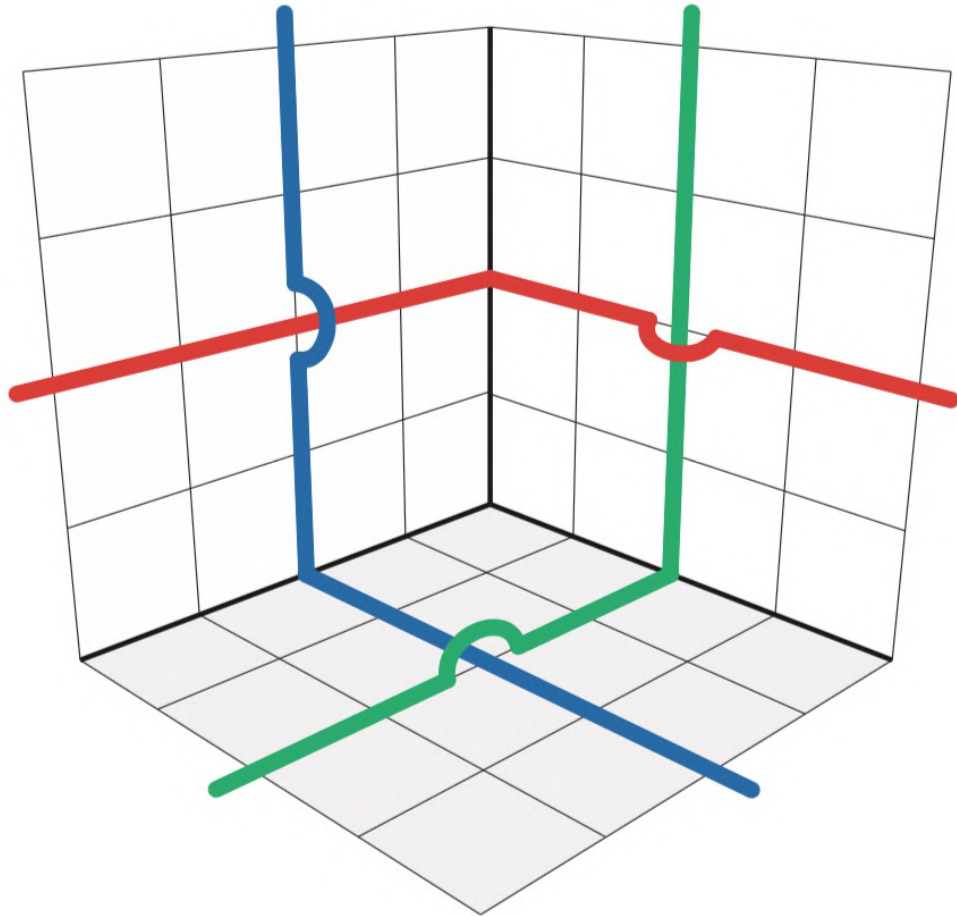
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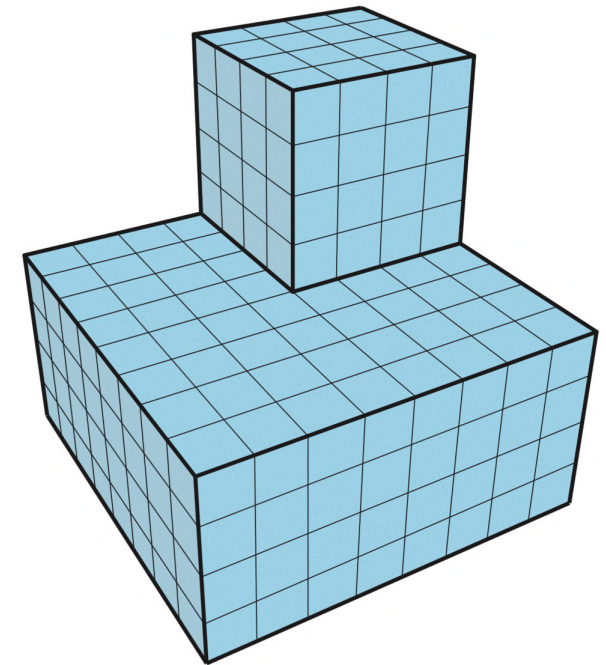
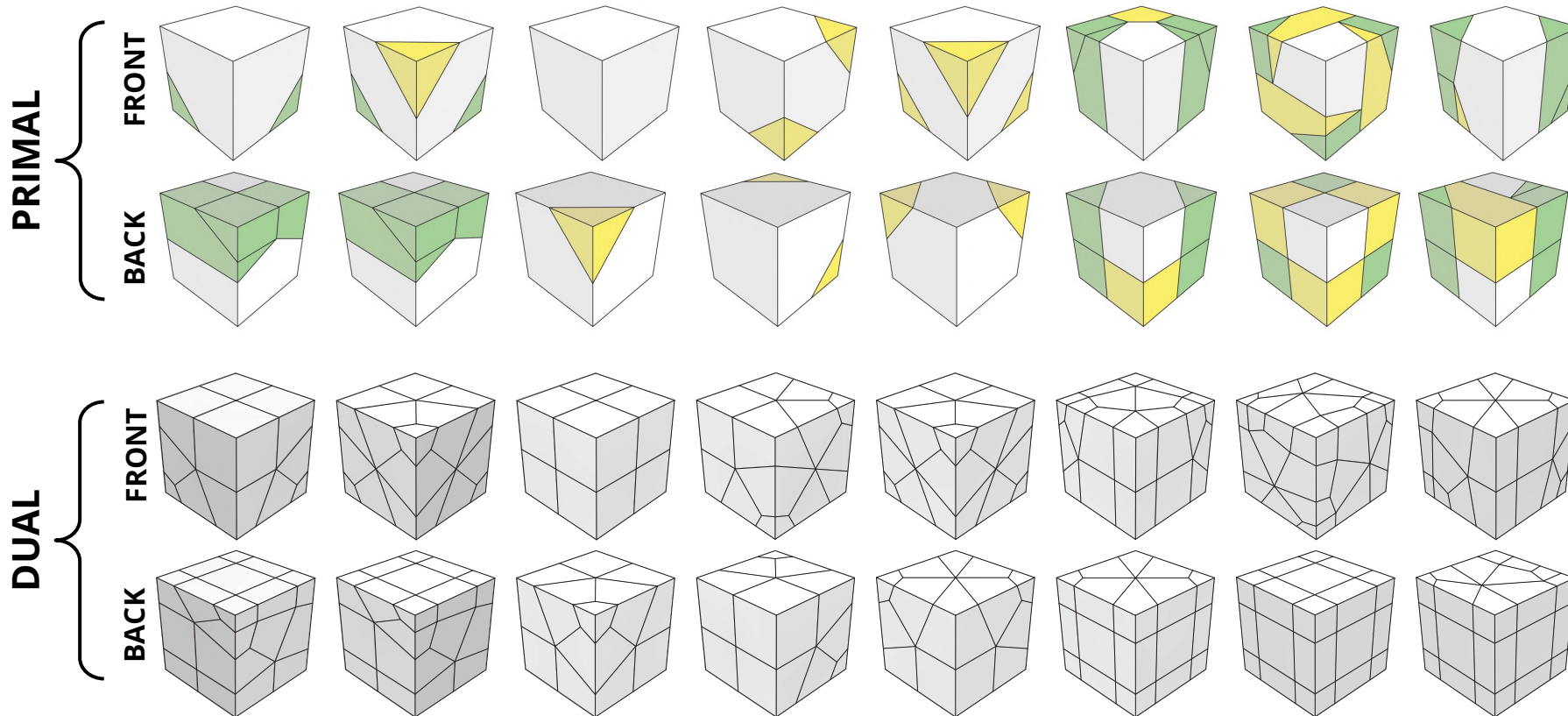
# Concave corners

There are two possible ways to handle a concave corner



# Reduced scheme set

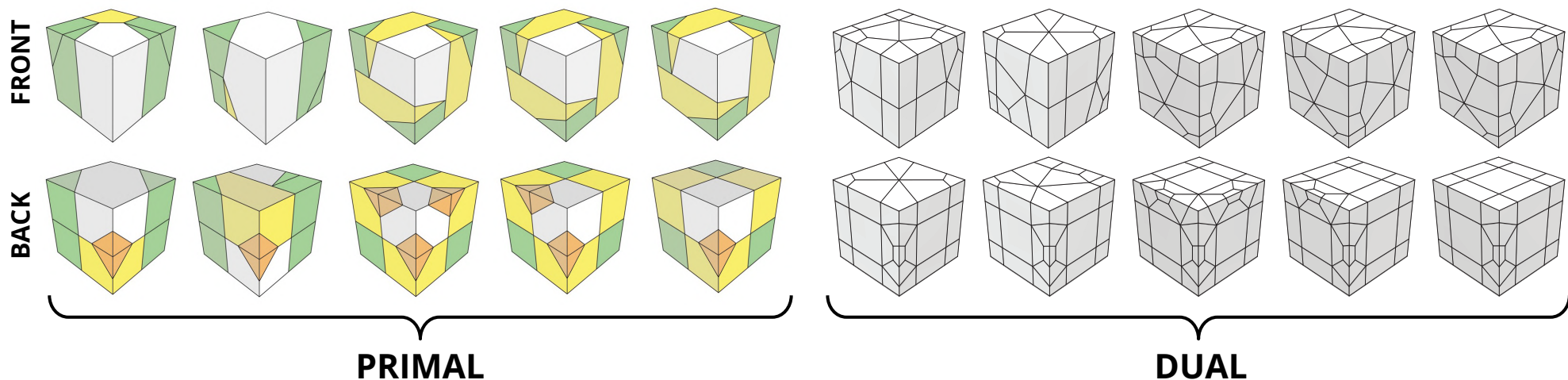
Eight ***atomic building blocks*** are sufficient to reproduce all 20 possible transitions



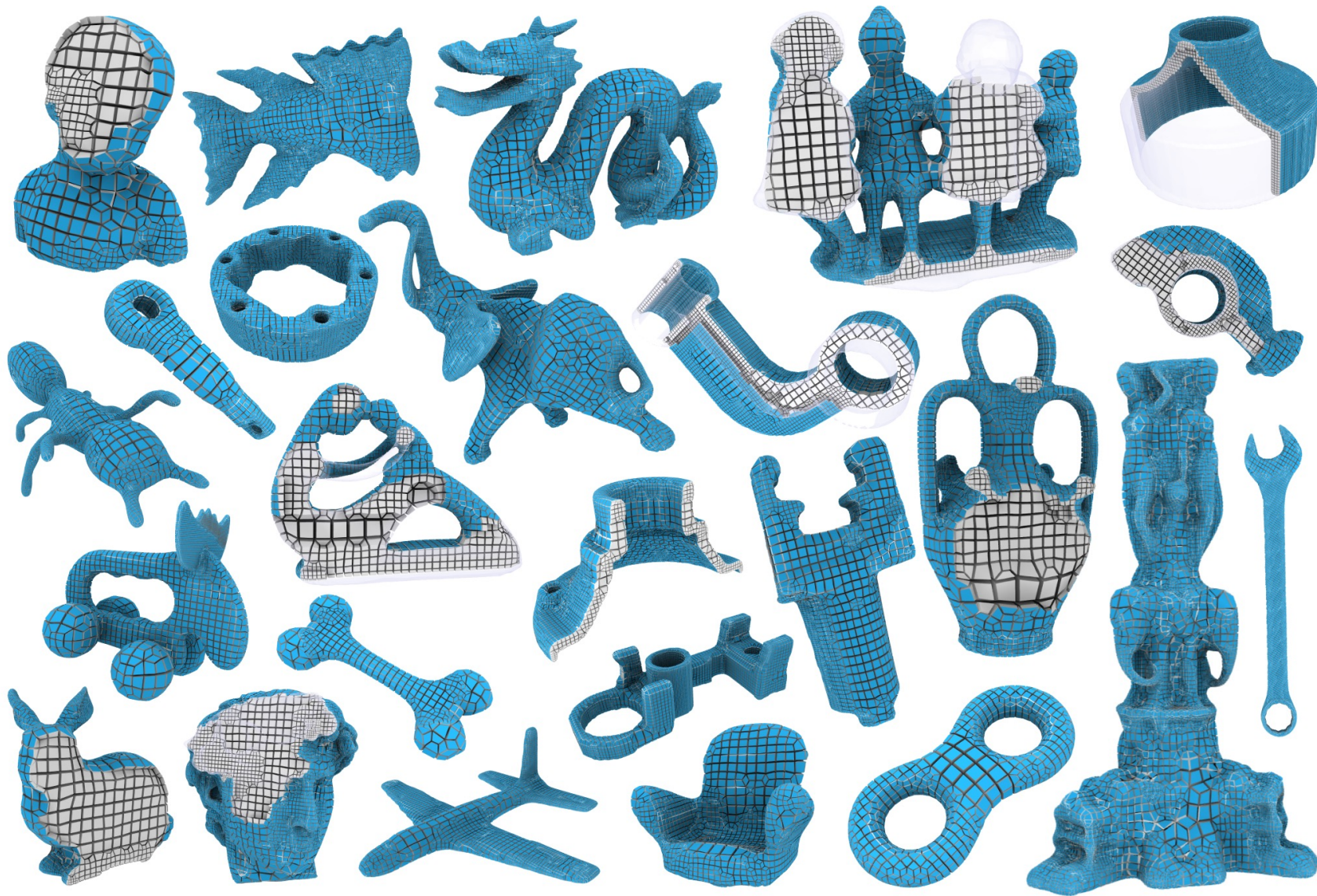
# Extension to Weak Balancing

Balancing requires that adjacent cells differ by at most one level of refinement

- **Strong** balancing applies to face/edge/vert adjacent cells
- **Weak** balancing applies to face adjacent cells only
- ▶ **PRO:** achieve balancing with much less refinement
- ▶ **CON:** additional schemes for hybrid convex/concave edges

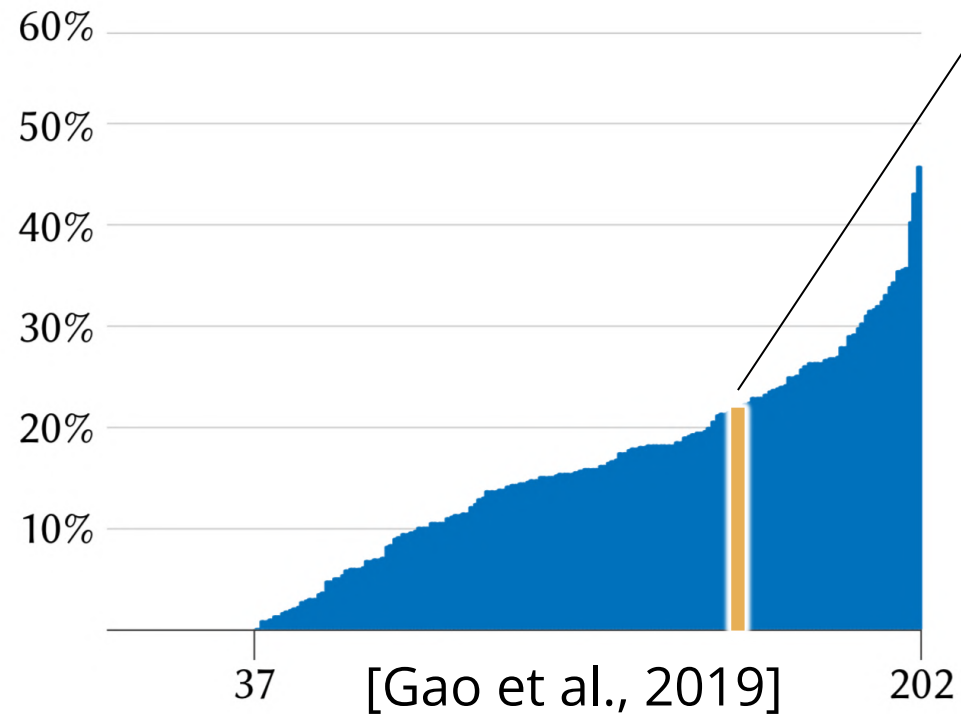
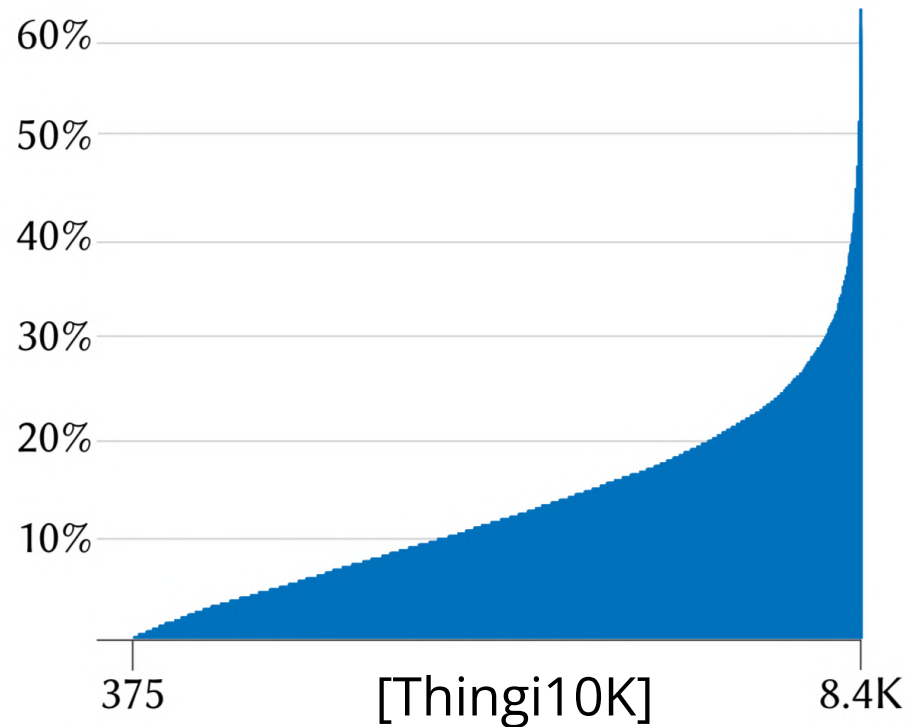


## Validation, Analysis, Results



# Strong vs Weak Balancing

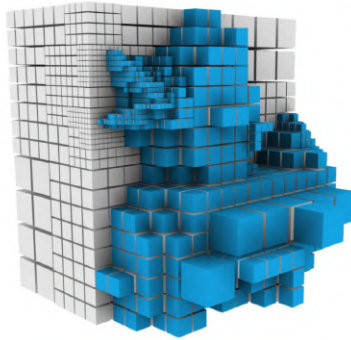
- **Input Grid:** octree
- **Refinement Policy:** local thickness (SDF)
- **Grid Pairing:** octree



NORMALIZED RATIO

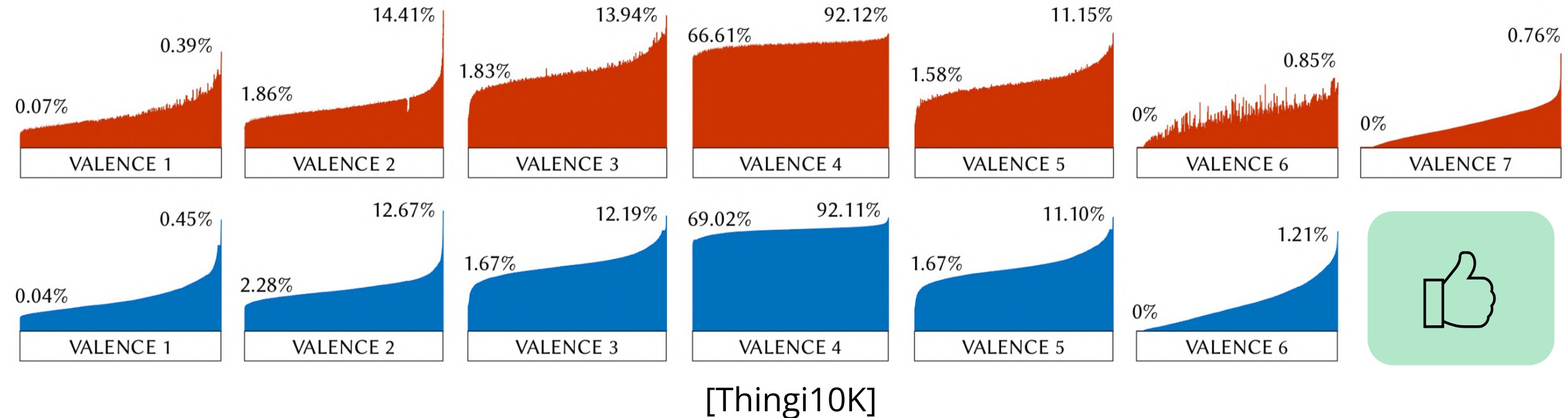
$$\frac{|H_S| - |H_W|}{|H_W|}$$

\* HS = cells with Strong balancing  
HW = cells with Weak balancing



# Edge valences

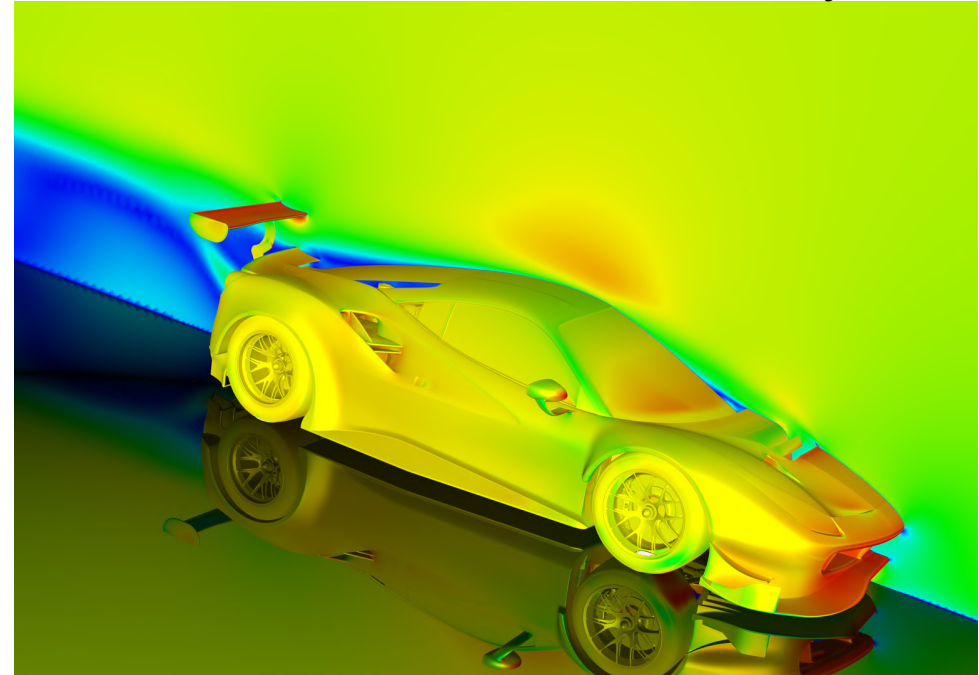
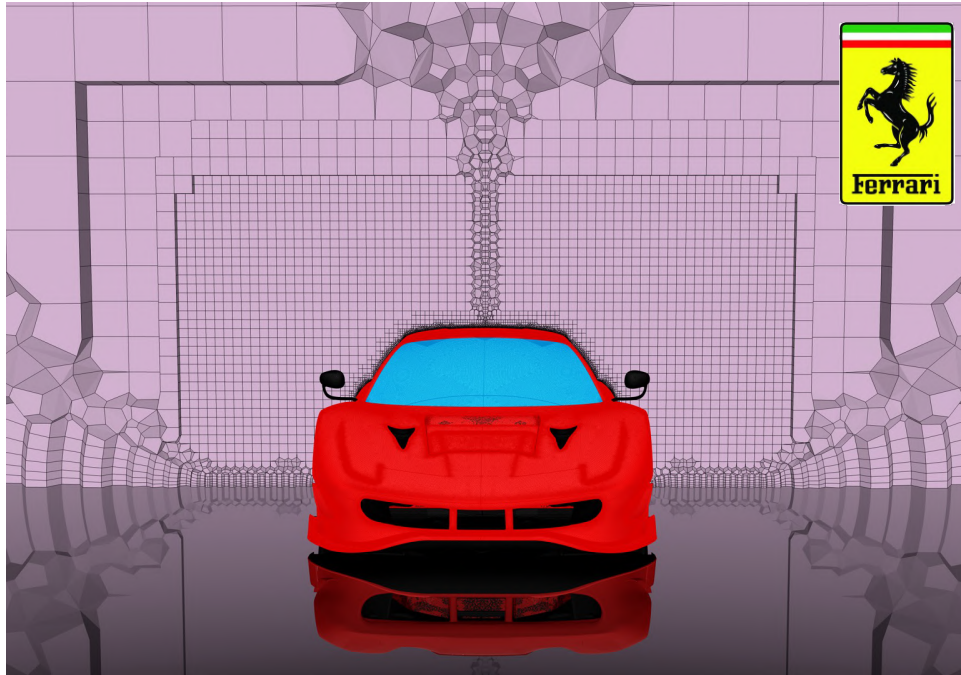
**Optimal** transition schemes, i.e., they handle intersections of prismatic elements to minimize per edge valence



[Gao et al., 2019] fails in 37 models (not considered in the chart)

# Are these improvements important?

[Ferrari 488 GTE (May 2020)]



"Ferrari engineers used Mosaic enabled Poly-Hexcore meshing technology to **cut the number of cells by 15%**. This reduction [...] slashes overall solve times by half. The Ferrari team can now run **300% more CFD simulations**, helping the engineers to develop their cars or new solutions faster"



# Conclusions

**4**  
Main contributions

EXHAUSTIVITY

AMBIGUITY

OPTIMALITY

WEAK BALANCING

PUBLIC CODE

+

ONLINE DEMO



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Thanks!



MORE INFO