SIGGRAPH ASIA 2021 TOKYO
LIVE

# Generalized Adaptive Refinement for Grid-based Hexahedral Meshing

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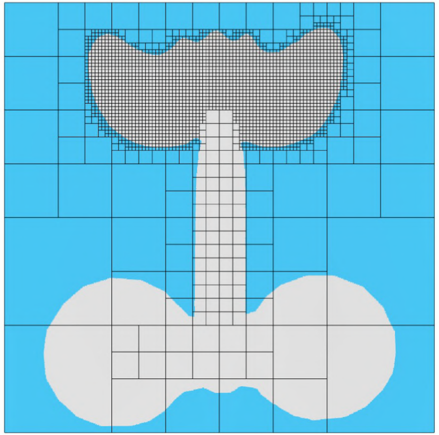
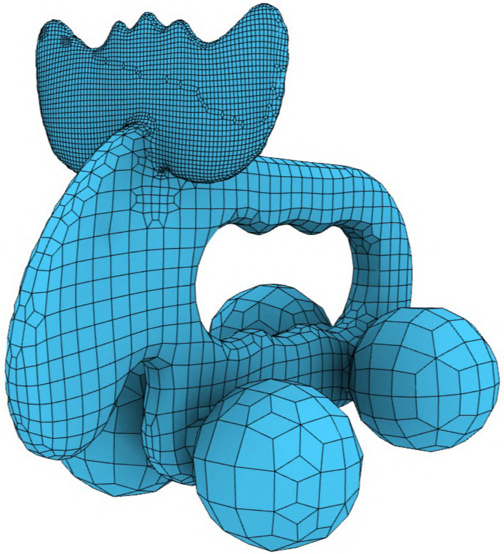
[2] CRS4, Italy

[3] CNR-IMATI, Italy

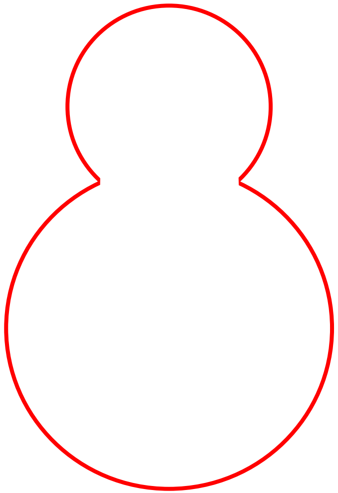
adaptive grid



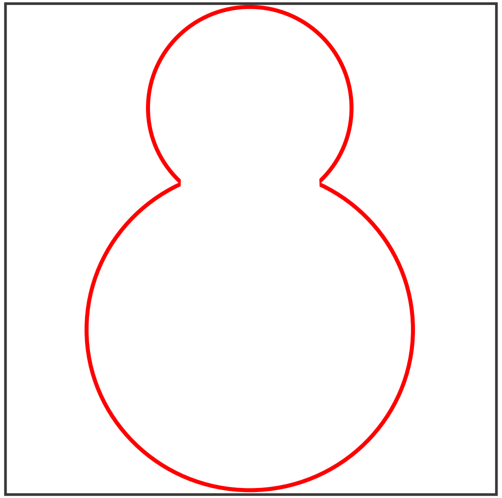
conforming hexmesh



# Grid-based Hexmeshing

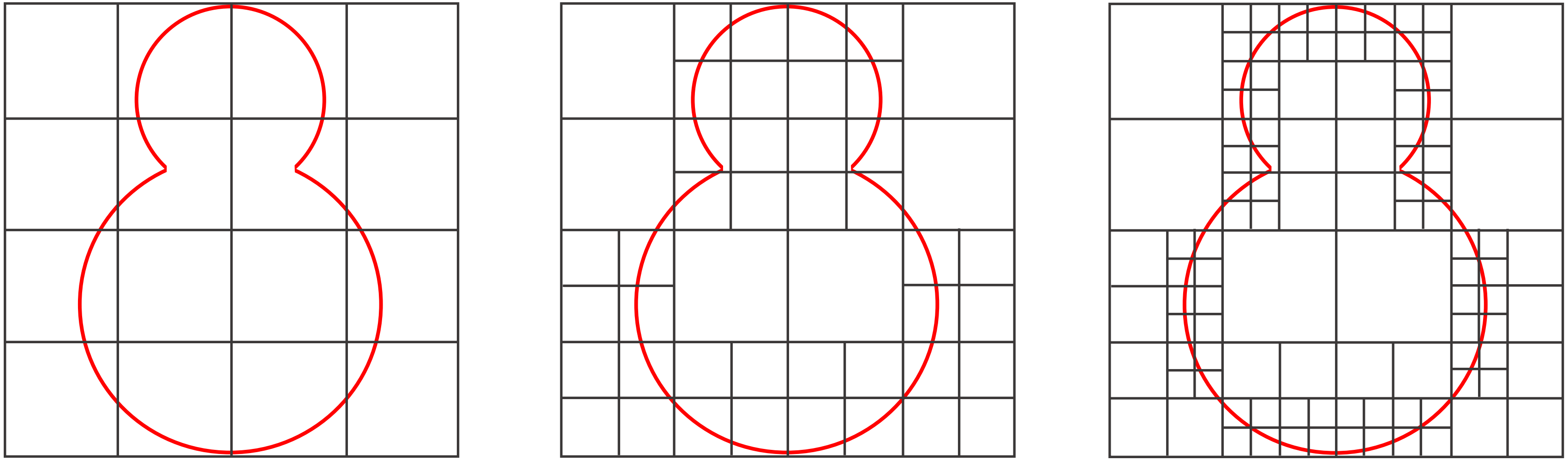


start from an
input surface



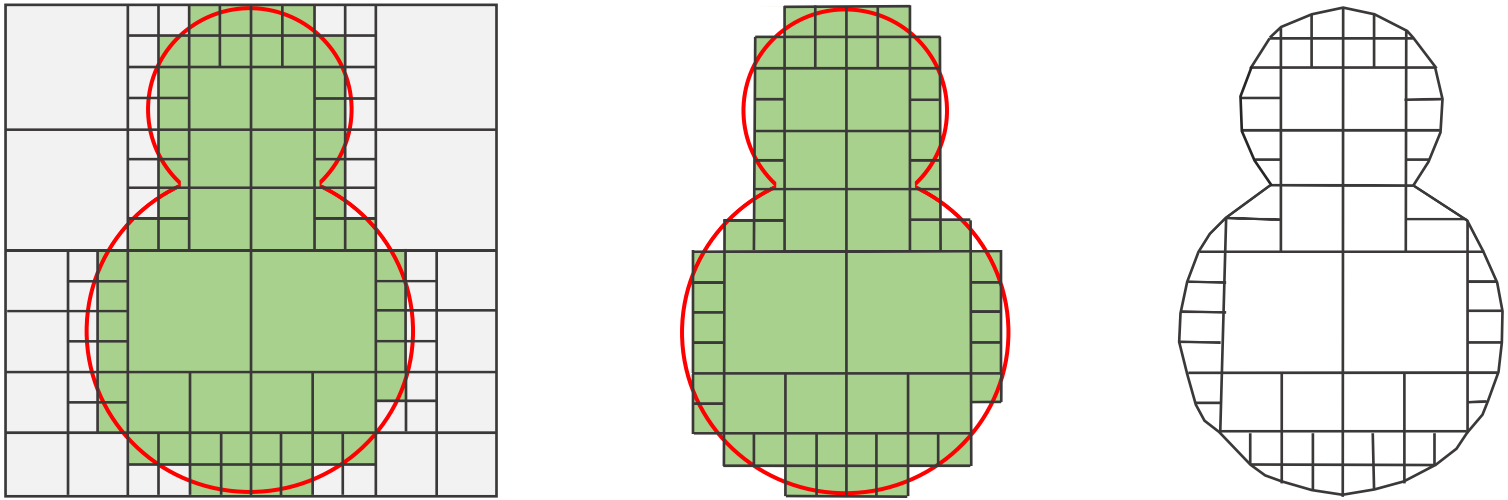
fit the input surface
on its AABB

# Grid-based Hexmeshing



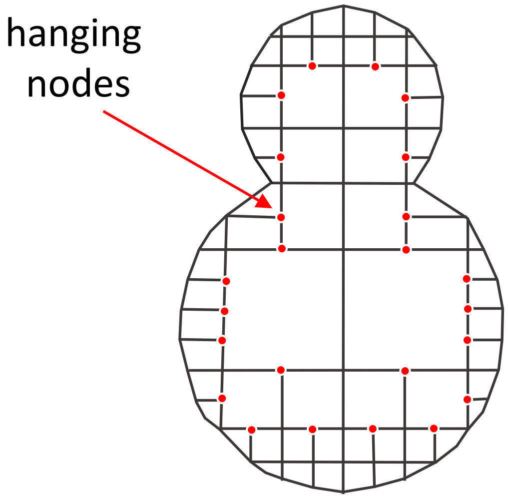
refine the AABB according to a metric

# Grid-based Hexmeshing



discard useless hexaedra and project the grid on the target surface

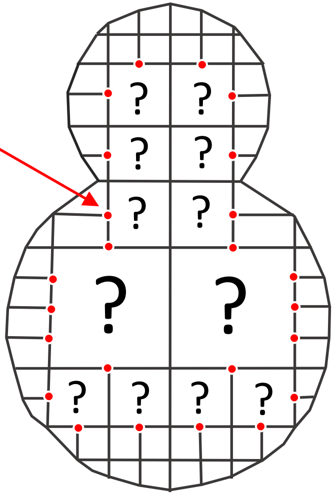
# Grid-based Hexmeshing




a non-conforming mesh
with hanging nodes
is obtained

# Grid-based Hexmeshing

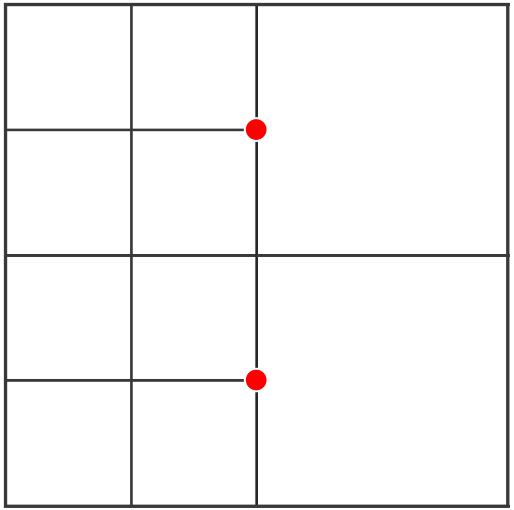
hanging
nodes



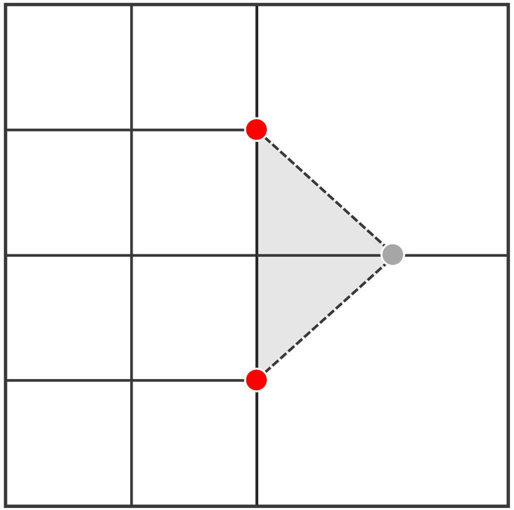

a non-conforming mesh
with hanging nodes
is obtained

goal: remove hanging nodes!

# Remove hanging nodes



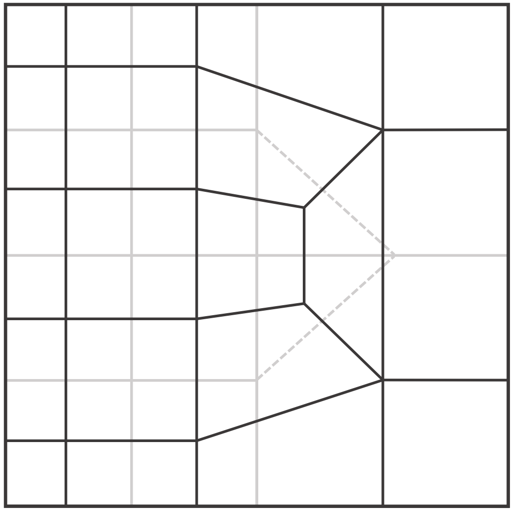
# Remove hanging nodes



# Remove hanging nodes

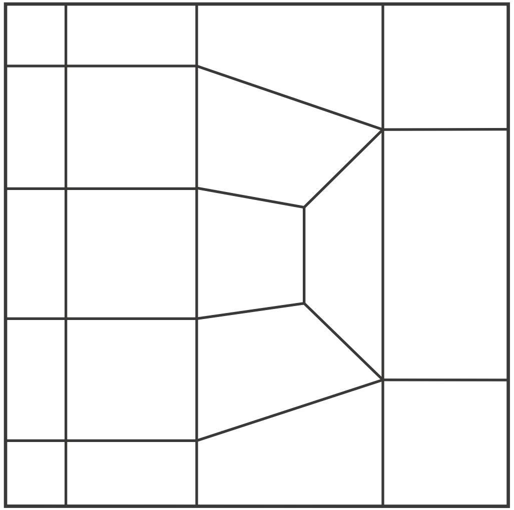


dualization
is required!



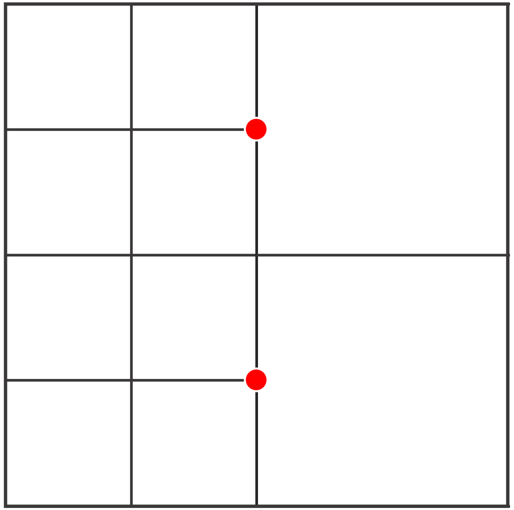
# Remove hanging nodes





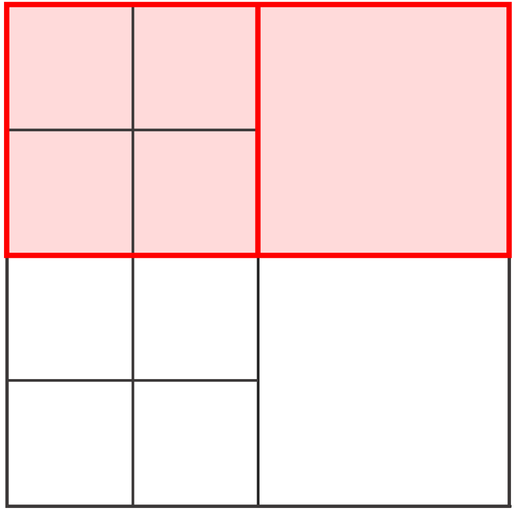
mission complete:
conforming hexmesh

# Remove hanging nodes



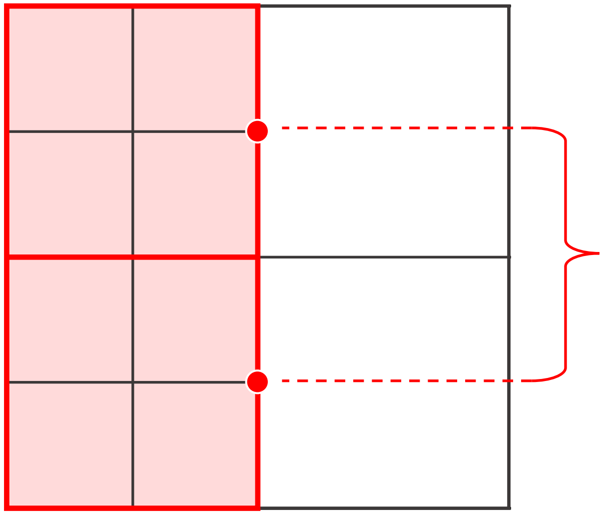
# Remove hanging nodes

**Balancing:**
the difference in the amount of refinement between adjacent cells cannot be greater than 1



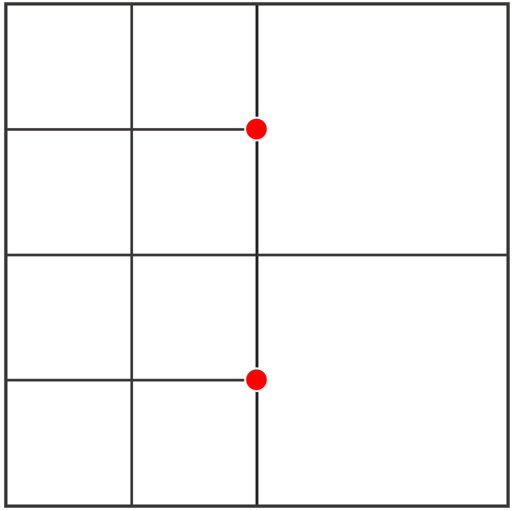
# Remove hanging nodes

**Balancing:**
the difference in the amount of refinement between adjacent cells cannot be greater than 1

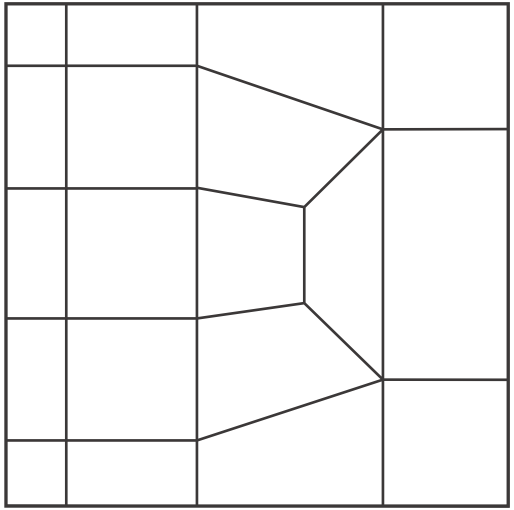


**Pairing:**
hanging nodes must be taken in pairs to be removed

# Remove hanging nodes

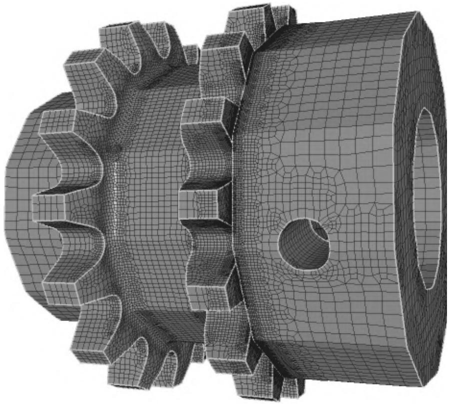


non conforming
grid

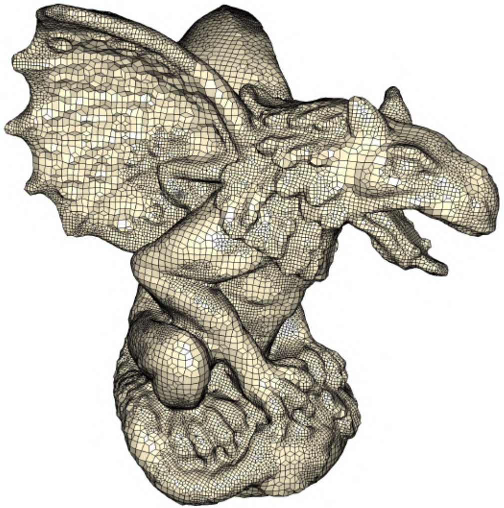


conforming
hexmesh

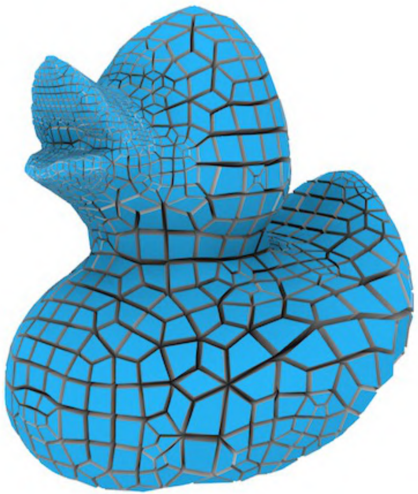
# State of the art



[Maréchal 2009]



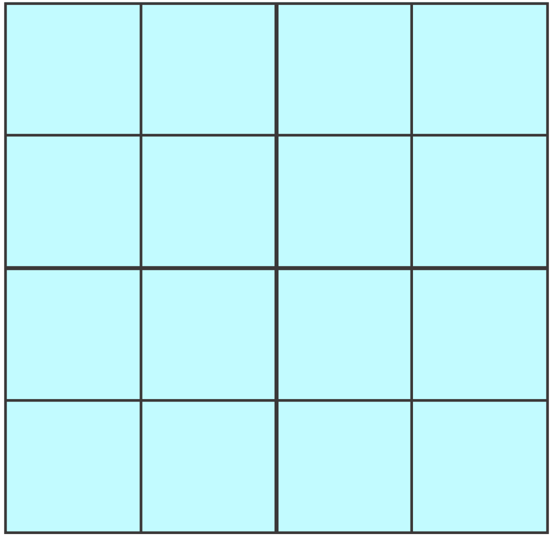
[Gao et al. 2019]

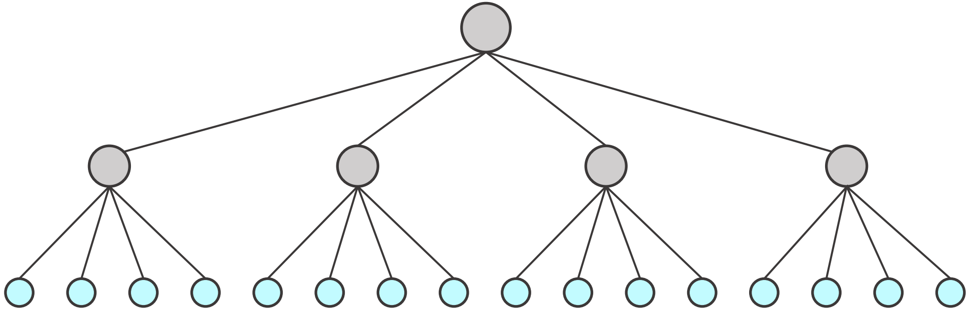


[Livesu et al. 2021]

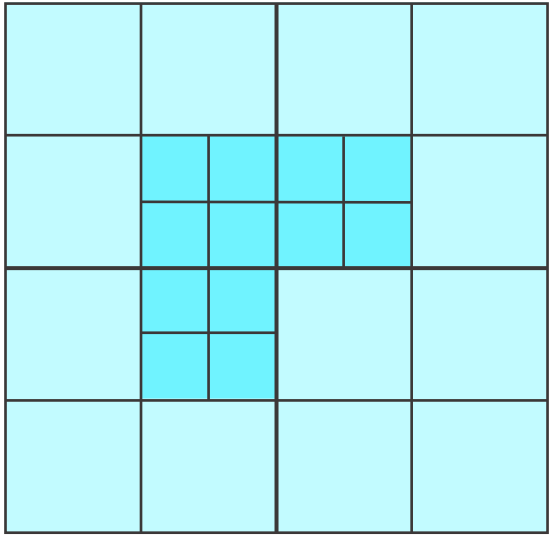
# Pairing in Octree grids

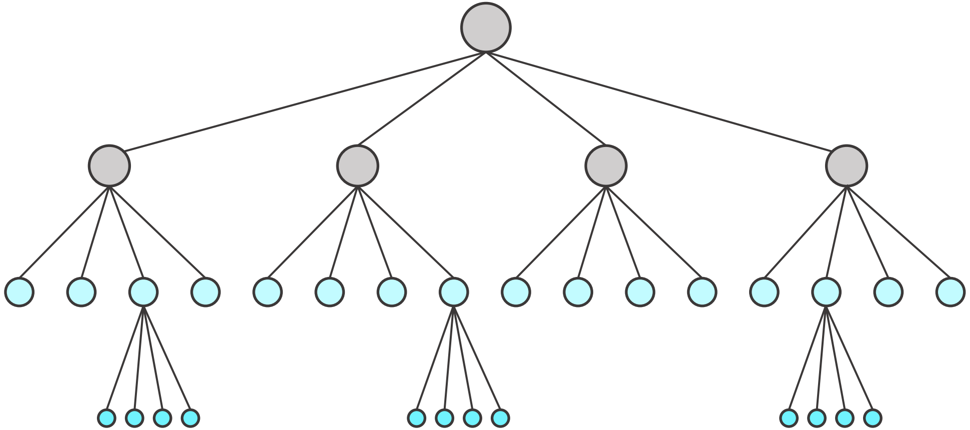






# Pairing in Octree grids

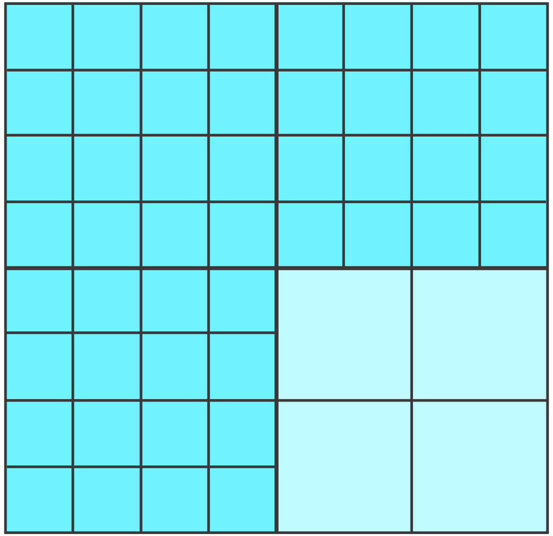




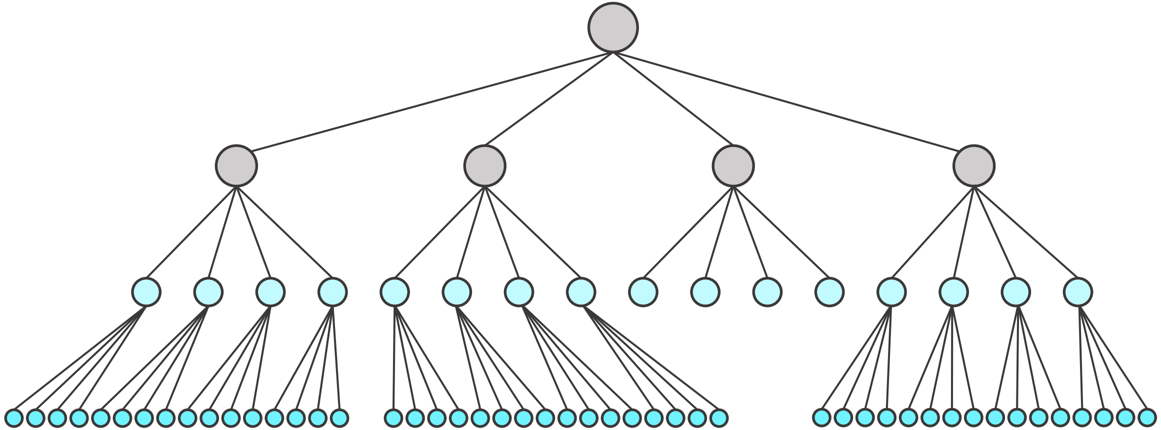
# Pairing in Octree grids



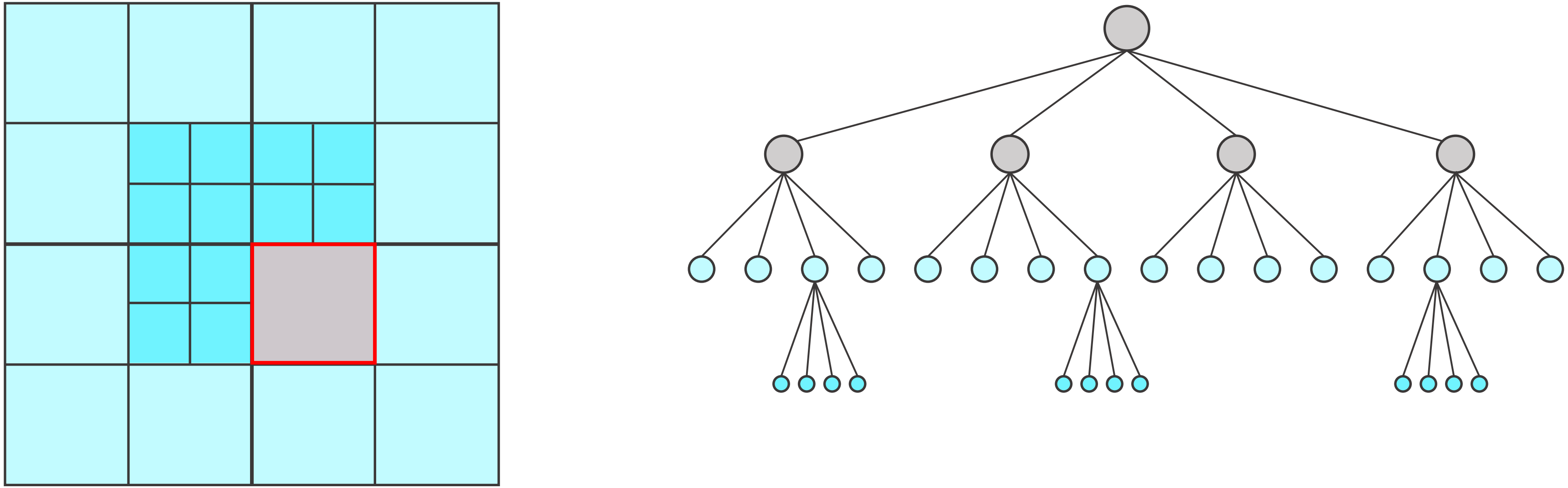
grid pairing



tree pairing

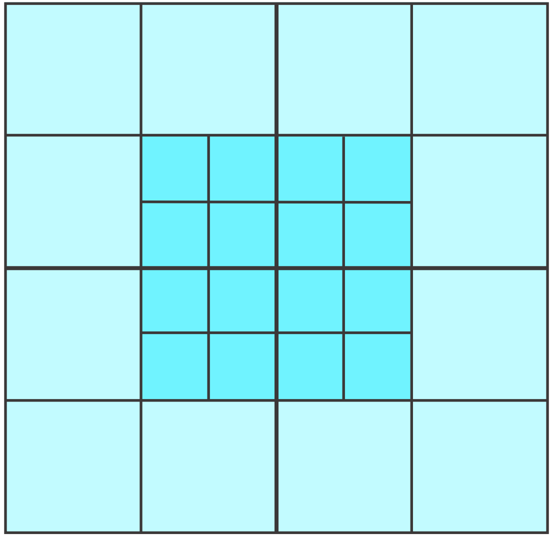


# Pairing in Octree grids

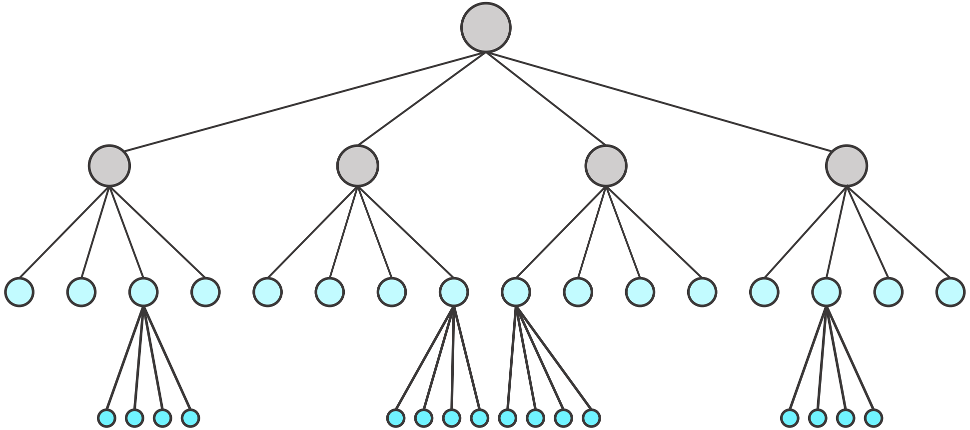


# Pairing in Octree grids

grid pairing



NO tree pairing

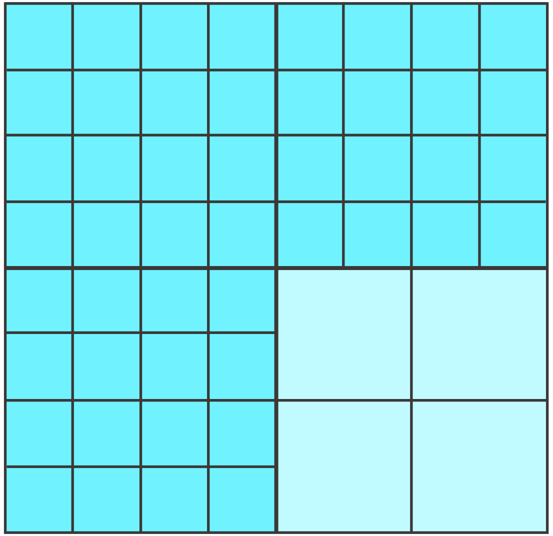


# Tree pairing vs NO our pairing

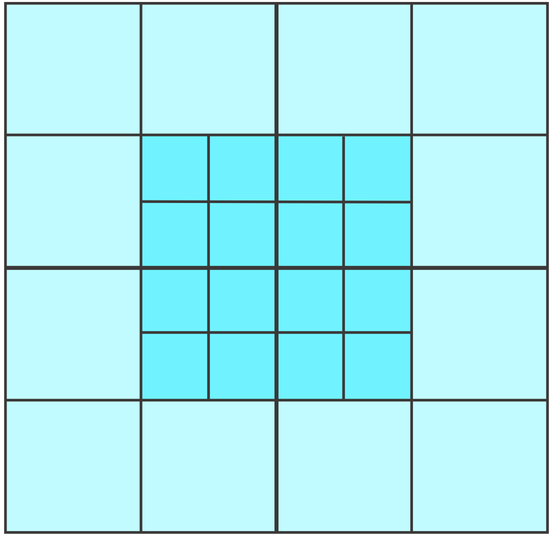
Num cells

25⟶52

too much
over-refinement



State-of-the-art
tree pairing



our
pairing

Num cells

25⟶28

less
over-refinement!

# Our Contribution

# Contributions

- Refinement on vertices
- Guarantee the pairing condition through an ILP
  - Linear objective function
  - All linear constraints
- Get rid of the tree structure constraints for balancing and pairing

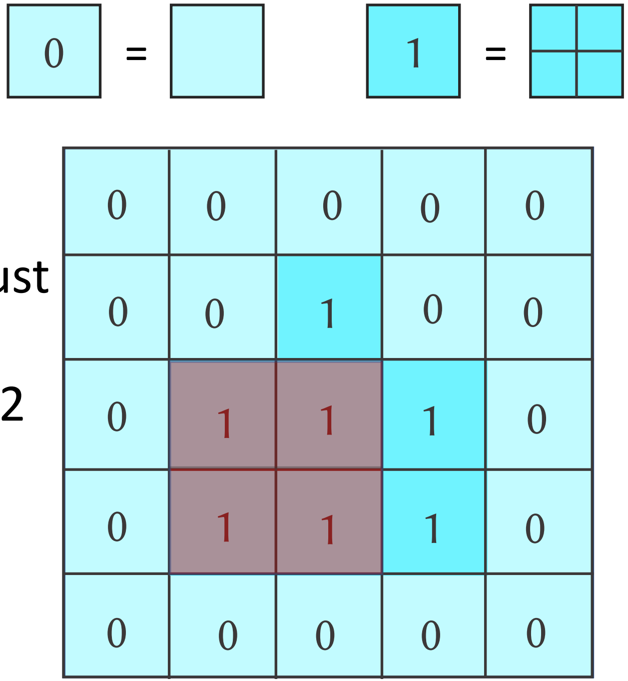
# ILP Formulation for Regular Grids


binary grid

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |

# ILP Formulation for Regular Grids

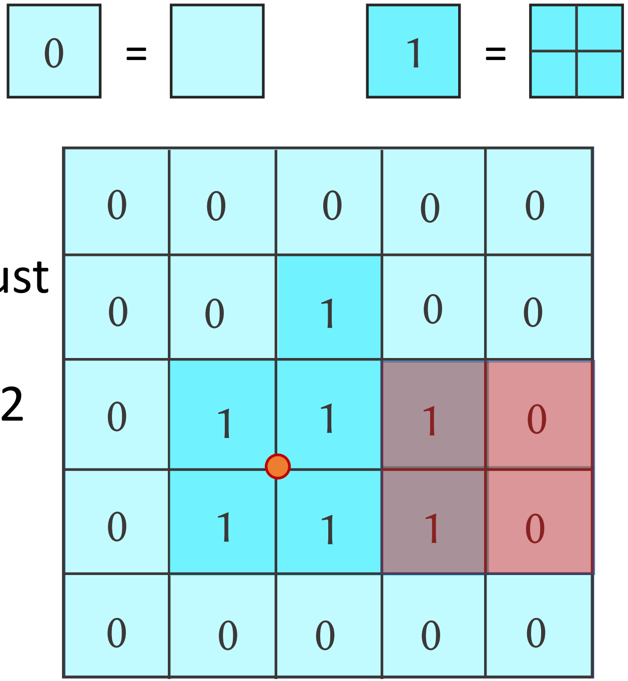





every refined cell must fit into a non-overlapping 2x2 mask

# ILP Formulation for Regular Grids



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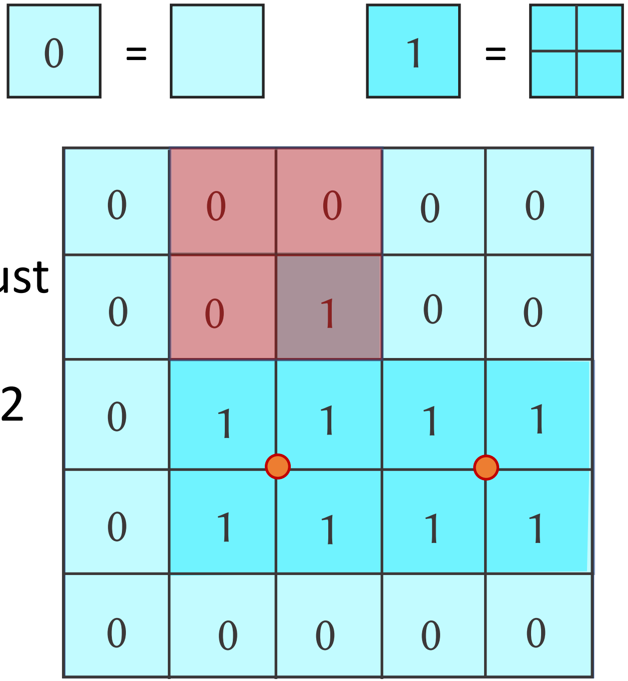
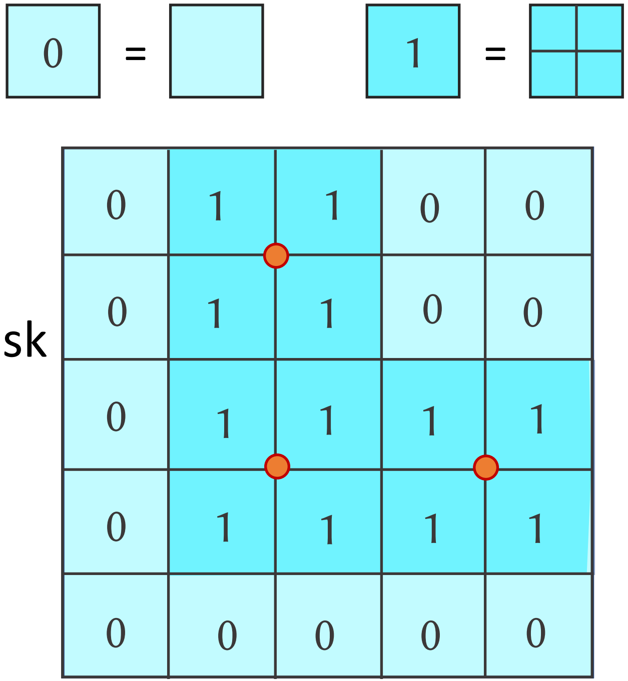



every refined cell must fit into a non-overlapping 2x2 mask

# ILP Formulation for Regular Grids






cells touching
the center of the mask
are refined

# ILP Formulation for Adaptive Grids

$$\boxed{\min_{r(v)} E = \sum_{c \in G} \left( \sum_{v \in c} r(v) - r(c) \right)}$$

minimize the number of refined cells

$$s.t.$$

$$\forall c \in G, \quad \sum_{v \in c} r(v) \geq r(c)$$

$$\forall ij \in N_P \quad r(v_i) + r(v_j) \leq 1$$

# ILP Formulation for Adaptive Grids

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$$s.t.$$

$$\boxed{\forall c \in G, \quad \sum_{v \in c} r(v) \geq r(c)}$$

$$\forall ij \in N_P \quad r(v_i) + r(v_j) \leq 1$$

the refinement assigned to a cell must always be lesser or equal to the refinement assigned to its vertices

# ILP Formulation for Adaptive Grids

$$\min_{r(v)} E = \sum_{c \in G} \left( \sum_{v \in c} r(v) - r(c) \right)$$

$$s.t.$$

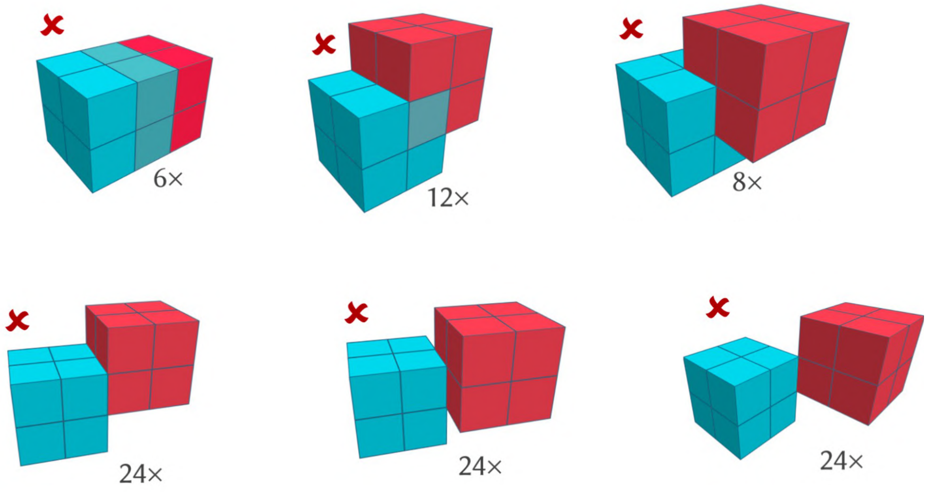
$$\forall c \in G, \quad \sum_{v \in c} r(v) \geq r(c)$$

$$\boxed{\forall ij \in N_P \quad r(v_i) + r(v_j) \leq 1}$$

two vertices whose minors overlap cannot be both refined

# Constraints

2x2x2 minors can't overlap → pairing guarantee



# ILP Formulation for Adaptive Grids

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$$s.t.$$

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Generalized Adaptive Refinement for Grid-based Hexahedral Meshing

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RICCARDO SCATENI, University of Cagliari, Italy

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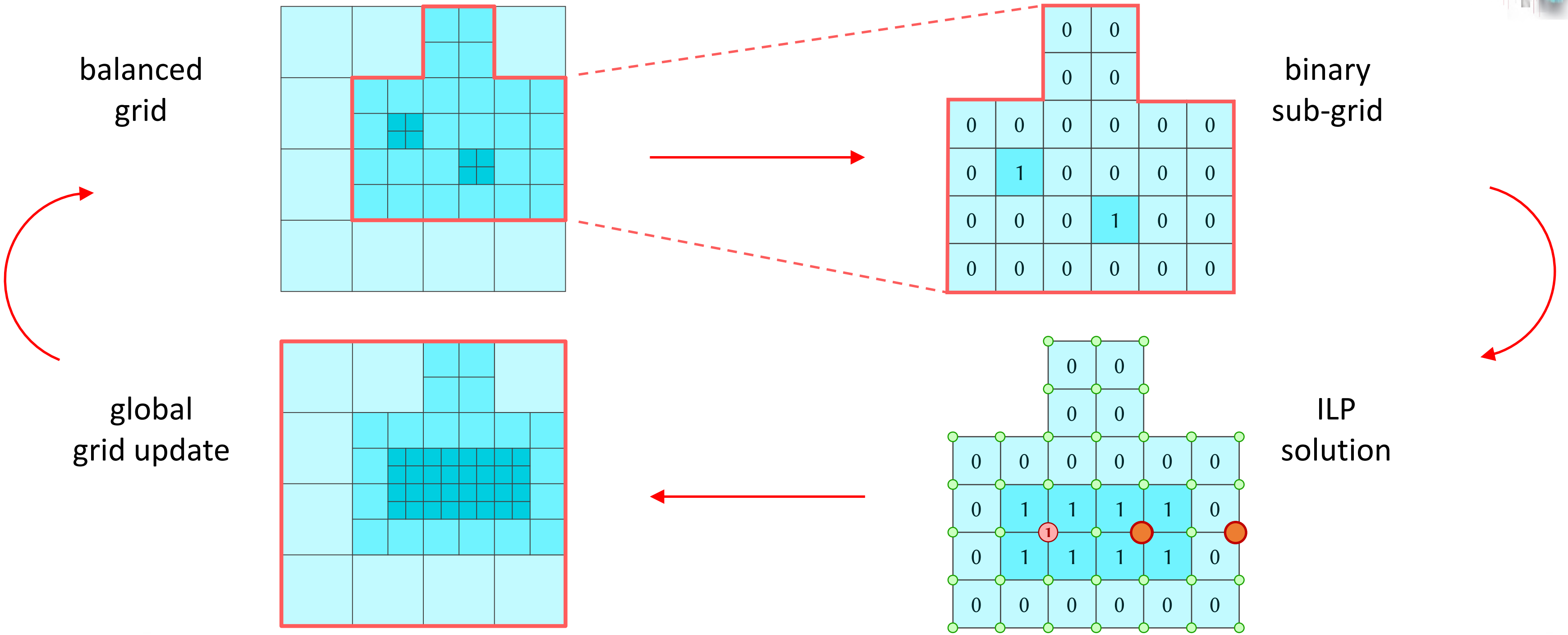

# Results

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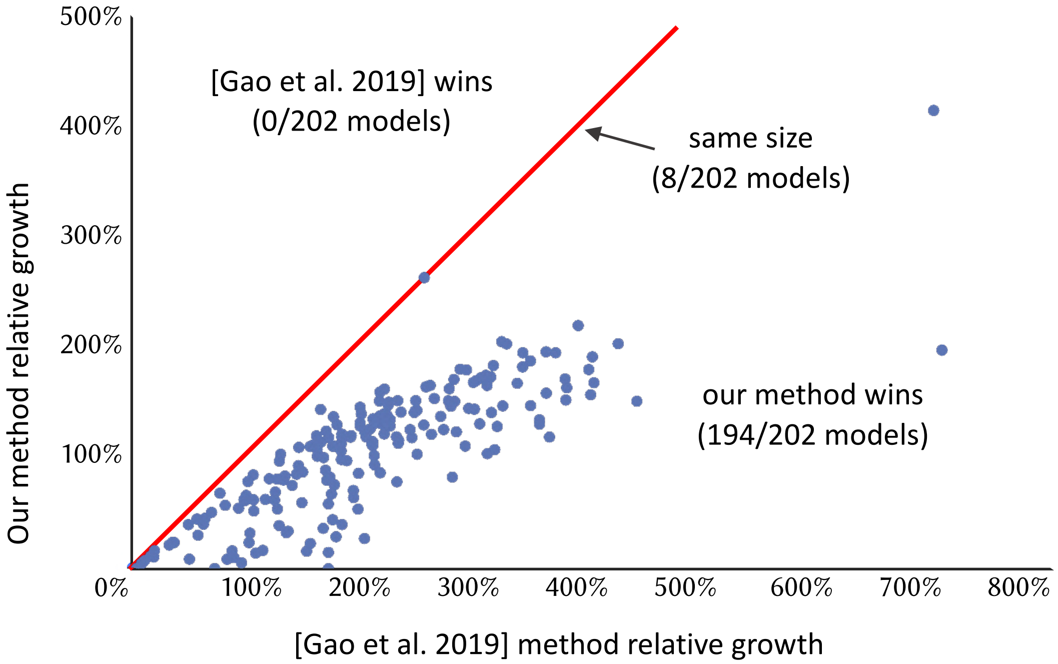
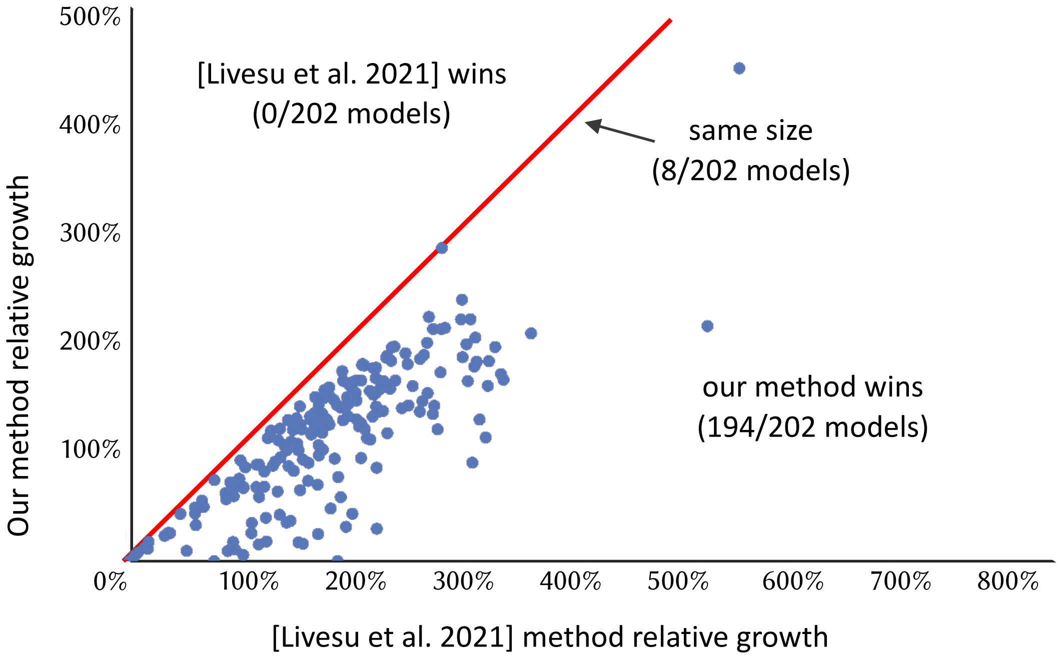



More than half the grid size if compared to [Gao et al. 2019]

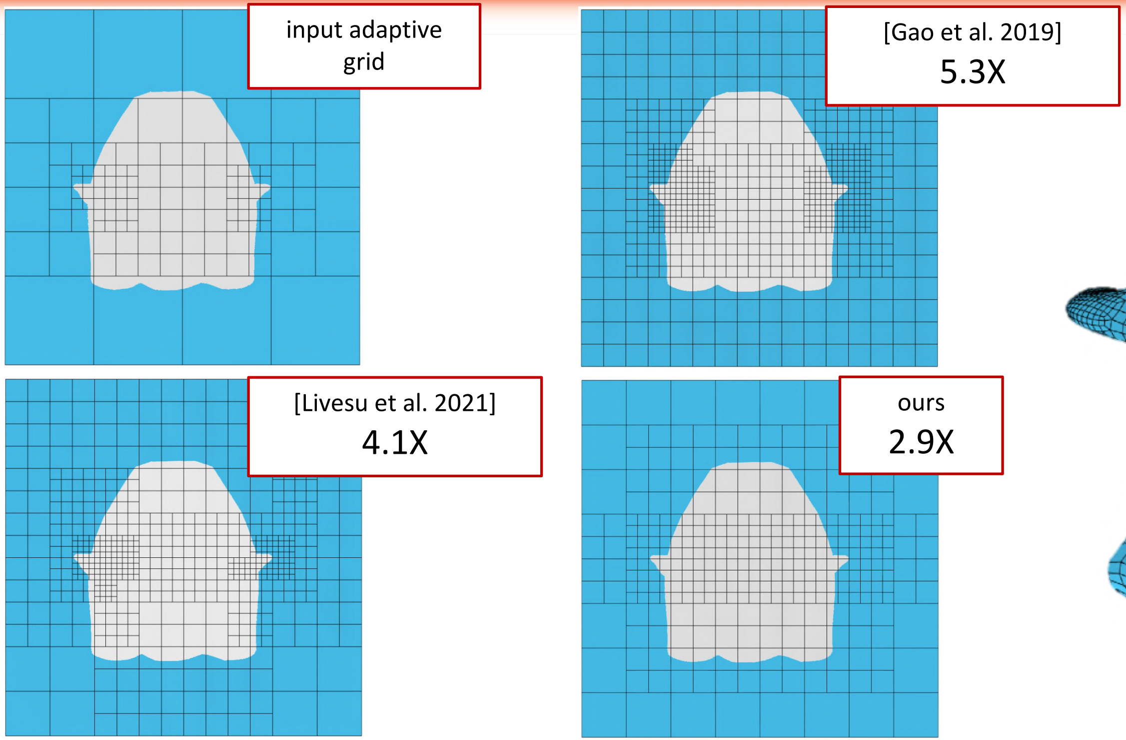
# Results

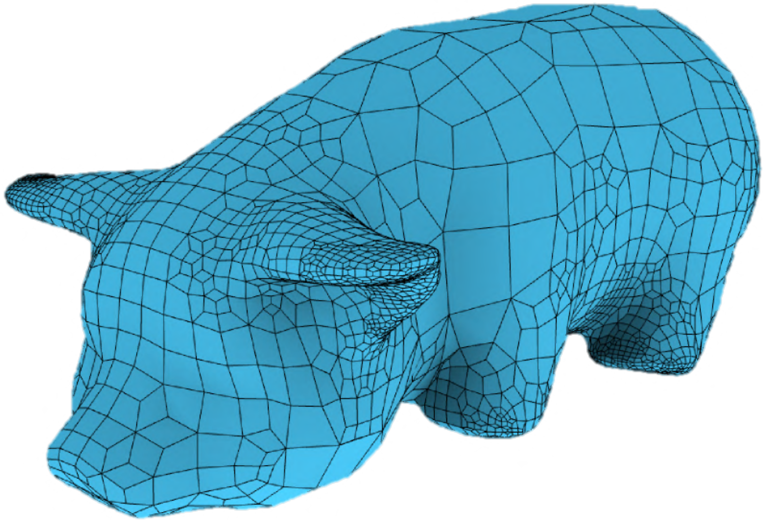



~ half the grid size if compared to [Livesu et al. 2021]!

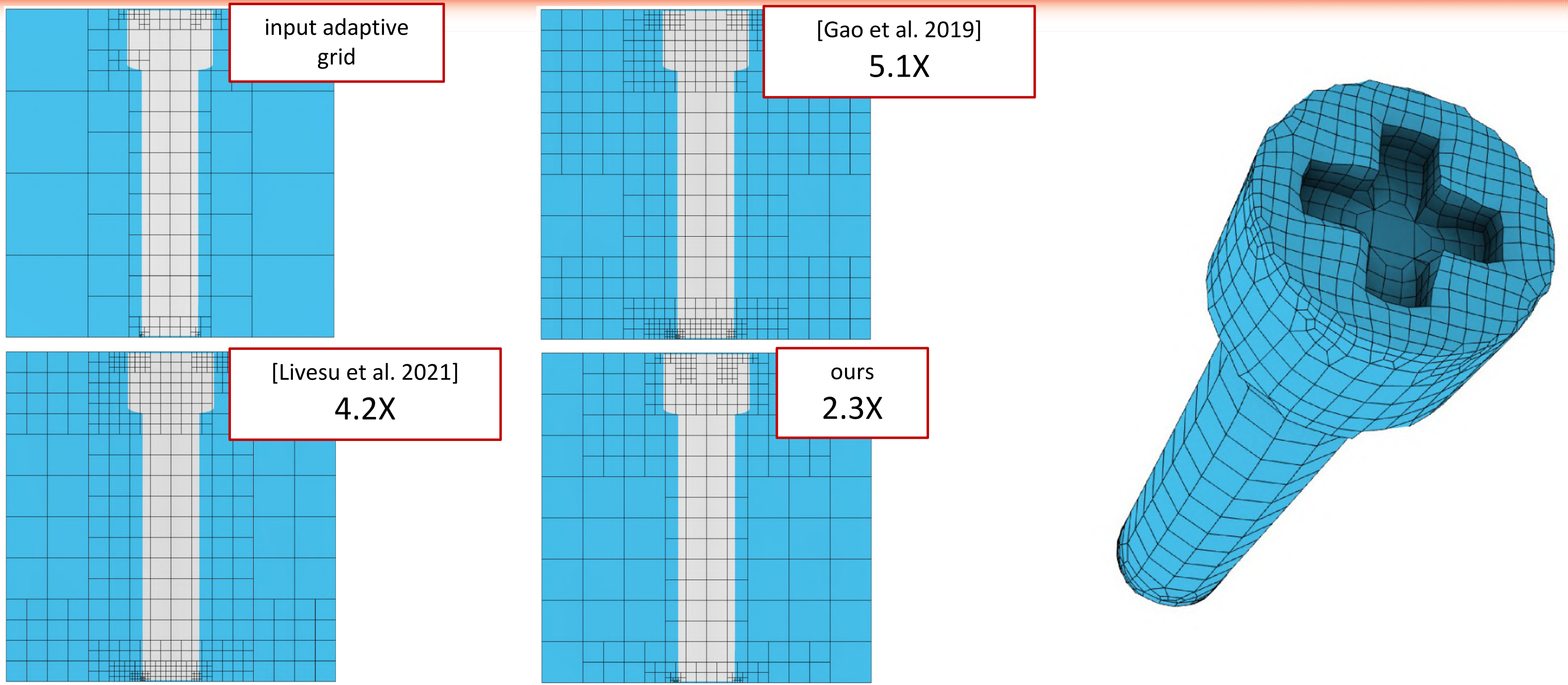
# Results



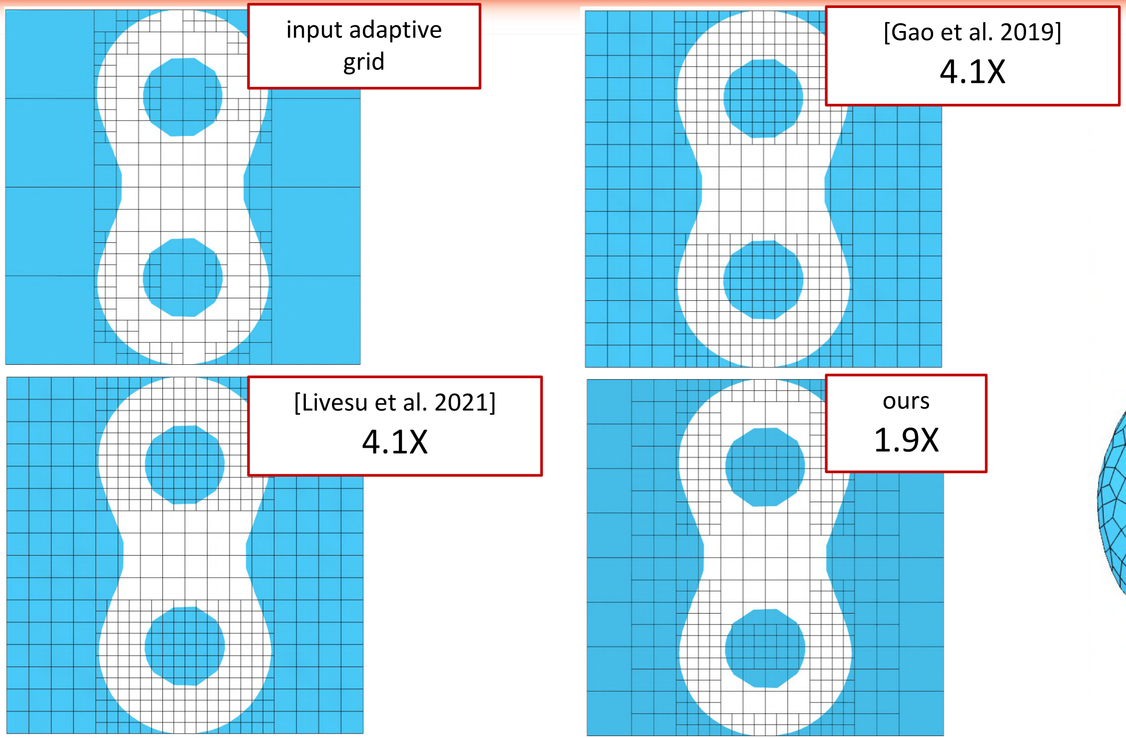

input adaptive grid
[Gao et al. 2019]
5.3X
[Livesu et al. 2021]
4.1X
ours
2.9X



# Results


input adaptive grid
[Gao et al. 2019]
5.1X
[Livesu et al. 2021]
4.2X
ours
2.3X

# Results



input adaptive grid
[Gao et al. 2019]
4.1X
[Livesu et al. 2021]
4.1X
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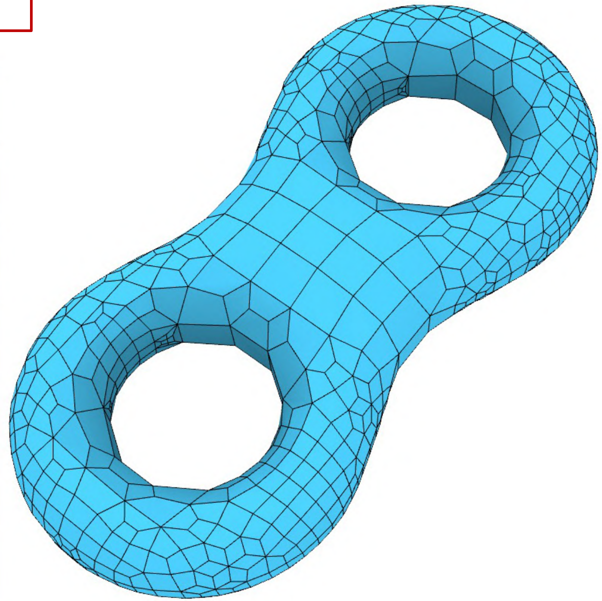
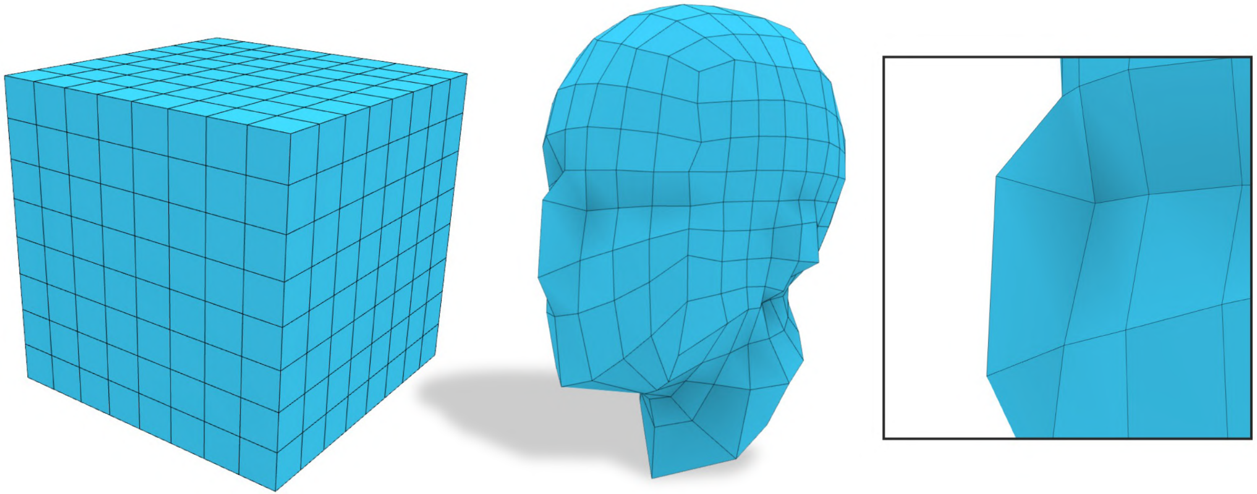
1.9X

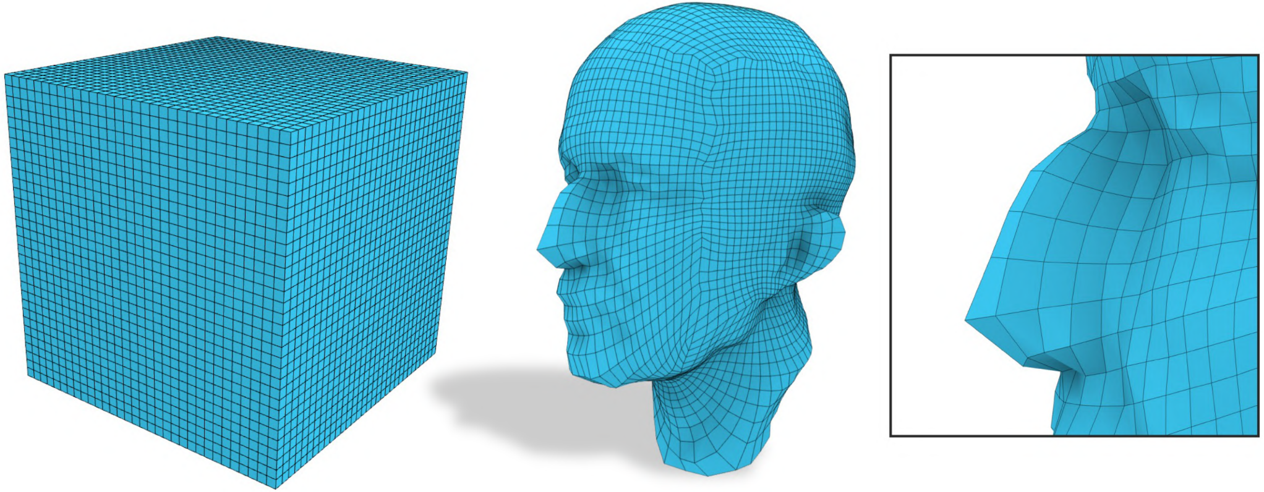



nose of Max Planck model collapsed on a face



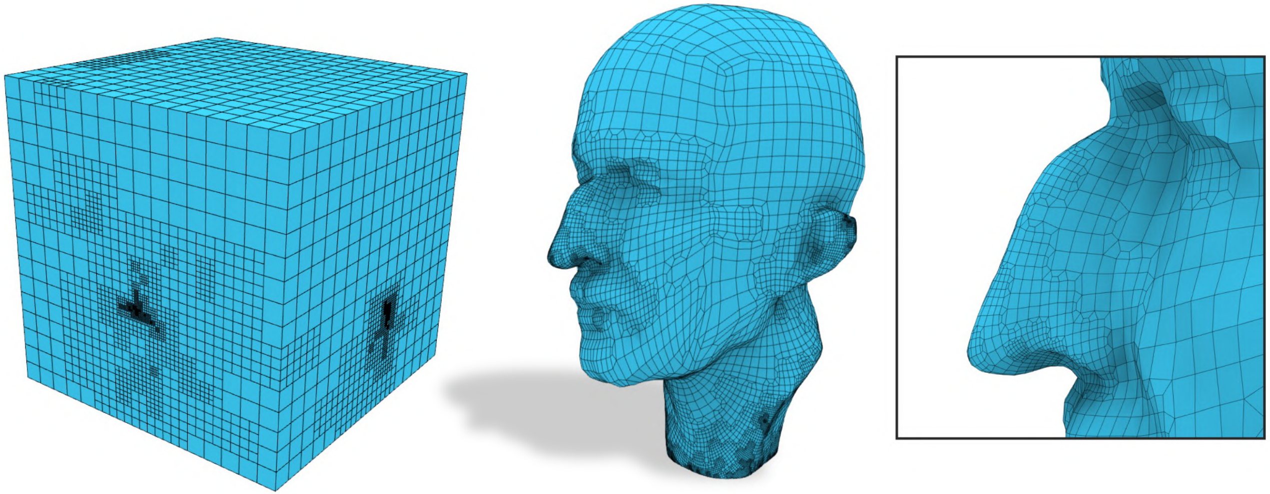
details are completely lost

nose of Max Planck model collapsed on a face



some details are preserved but at the cost of an excessive refinement

nose of Max Planck model collapsed on a face

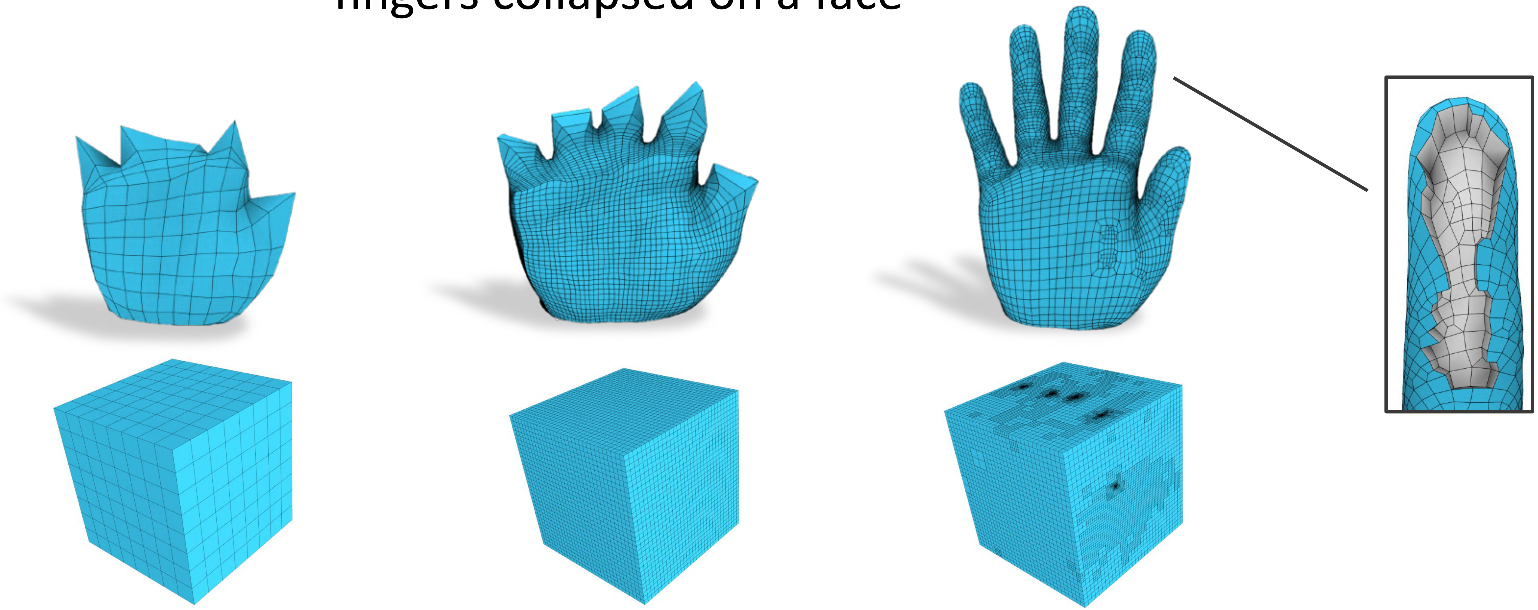


details are preserved

# Applications

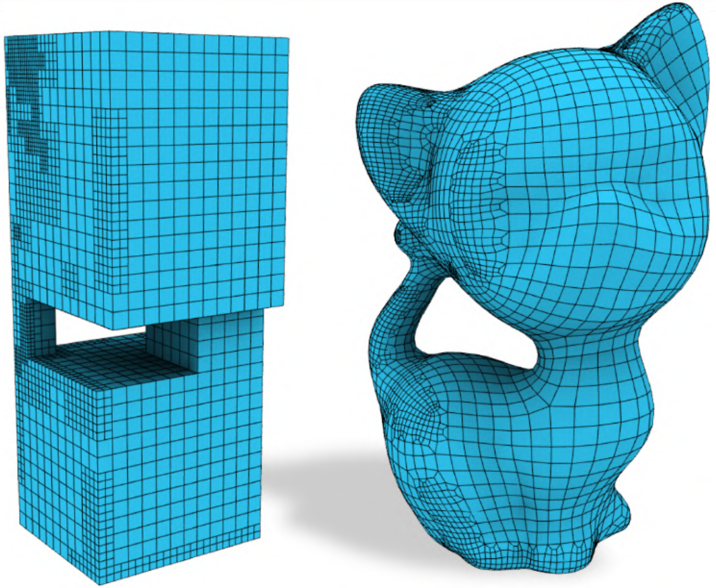


## fingers collapsed on a face



$8^3$ uniform grid | $32^3$ uniform grid | adaptive grid

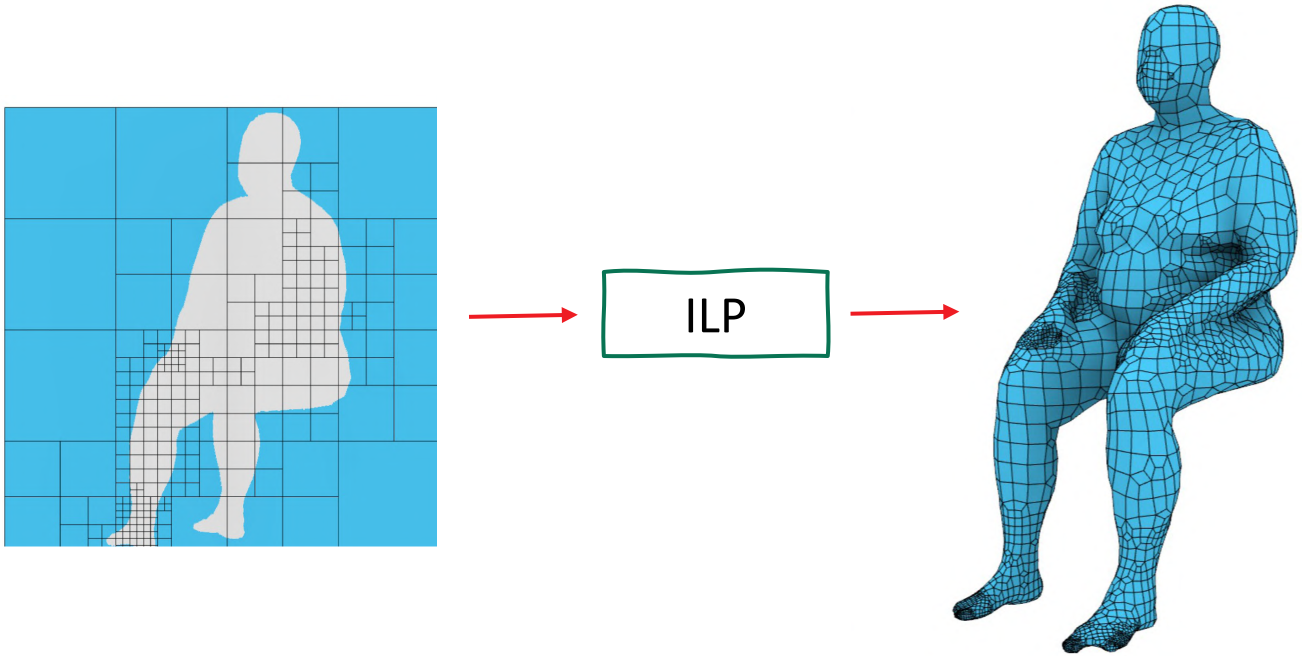
# Applications



not restricted to cube shapes

# Conclusion

# Conclusion



# Future Works

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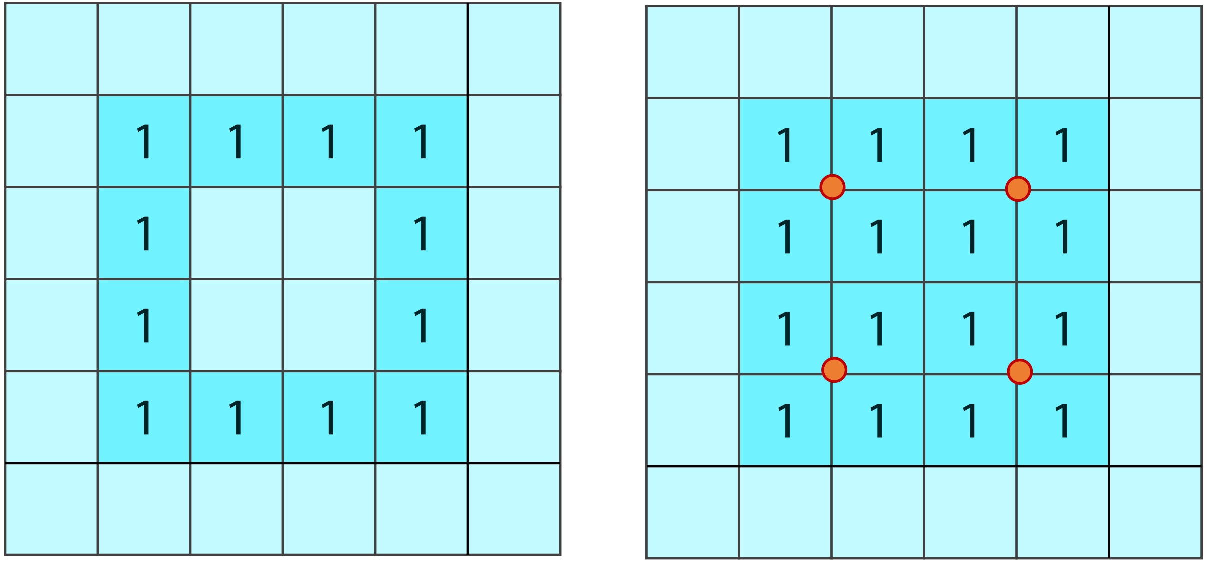
Input paired grid

Our output



# Code and Demo are available!

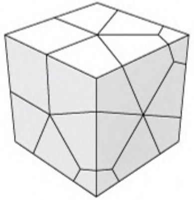


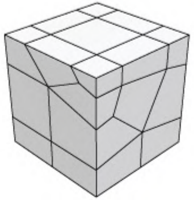
https://github.com/cg3hci/Gen-Adapt-Ref-for-Hexmeshing

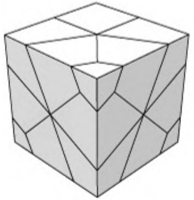
# Optimal Dual Schemes for Adaptive Grid Based Hexmeshing

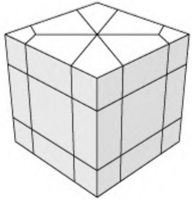
M. Livesu, L. Pitzalis, G. Cherchi

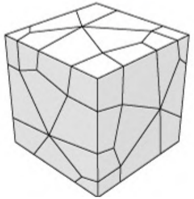
*ACM Transaction on Graphics 2021*

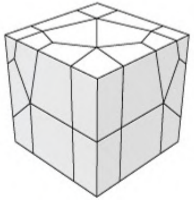












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CONFERENCE 14 - 17 DECEMBER 2021
EXHIBITION 15 - 17 DECEMBER 2021
TOKYO INTERNATIONAL FORUM, JAPAN

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# Thanks for your attention

Presented by

Luca Pitzalis